743.

Problem 39.83P (HRW)

A 1500 kg car moving at 20 m s⁻¹ approaches a hill that is 24 m high and 30 m long. We have to find the probability that the car will tunnel quantum mechanically through the hill, appearing on the other side. That is we have to find the car's transmission coefficient for this <u>hill</u>.

Solution:



The quantum transmission probability for a particle of mass *m*, kinetic energy *E* for tunnelling through a potential barrier of height *U* and width *L* is approximately given by the following function $T \approx \exp(-2kL)$,

where

$$k = \sqrt{\frac{8\pi^2 m (U - E)}{h^2}}$$

Mass of the car, m = 1500 kg,

speed of the car, $v = 20 \text{ m s}^{-1}$,

kinetic energy of the car,

$$E = \frac{1}{2}mv^{2} = \frac{1}{2} \times 1500 \times (20)^{2} = 3.0 \times 10^{5} \text{ J},$$

height of the hill, h = 24 m.

Therefore, the gravitational potential energy of the car in order to cross the hill will be,

$$U = mgh = 1500 \times 9.81 \times 24 = 3.53 \times 10^5 \text{ J}.$$

The width of the hill, L = 30 m.

Therefore, the quantum transmission probability for the car to tunnel through the will be approximately given by

the function

$$T \approx \exp(-2kL),$$

where

$$k = \sqrt{\frac{8\pi^2 m (U - E)}{h^2}} = \sqrt{\frac{8\pi^2 \times 1500 \times (0.53 \times 10^5)}{(6.63 \times 10^{-34})^2}}$$
$$= 1.19 \times 10^{38} \text{ m}^{-1}.$$

And

$$T = \exp(-2 \times 30 \times 1.19 \times 10^{38}) = \exp(-7.1 \times 10^{39}).$$