742.

<u>Problem 39.81P (HRW)</u>

Suppose a beam of 5.0 eV protons strikes a potential energy barrier of height 6.0 eV and thickness 0.70 nm, at a rate equivalent to a current of 1000 A. (a) We have to find the time on an average that we may have to wait for one proton to tunnel through the barrier. (b) We have to find the time on an average that we may have to wait for an electron instead of a proton to tunnel through the barrier.

<u>Solution:</u>

The quantum transmission probability for a particle of mass *m*, kinetic energy *E* for tunnelling through a potential barrier of height *U* and width *L* is approximately given by the following function

$T \approx \exp\left(-2kL\right),$

where

$$k = \sqrt{\frac{8\pi^2 m\left(U - E\right)}{h^2}}\ .$$

The data for the problem are as follows:

Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg ,

mass of an electron, $m_e = 9.11 \times 10^{-31}$ kg,

height of the potential energy barrier,

$U = 6.0 \text{ eV} = 9.6 \times 10^{-19}$ J,

kinetic energy of protons/electrons,

$E = 5.0 \text{ eV} = 8.0 \times 10^{-19}$ J,

and width of the potential barrier,

$L = 0.70 \text{ nm} = 7.0 \times 10^{-10}$ m.

(a)

The transmission probability for a proton to tunnel through the barrier will therefore be

$$T_{proton} \approx \exp\left(-2 \times 7.0 \times 10^{-10} \times \sqrt{\frac{8\pi^2 \times 1.67 \times 10^{-27} \times 1.0 \times 1.6 \times 10^{-19}}{\left(6.63 \times 10^{-34}\right)^2}}\right)$$

$$= \exp\left(-2 \times 7.0 \times 10^{-10} \times 2.19 \times 10^{11}\right)$$

$$= \exp\left(-306\right) = 10^{-132.8}.$$

It is given that the beam of protons/electrons is equivalent to a current of 1000 A. As the charge of proton is *e*, the number of protons striking the barrier per second will be

$$n = \frac{1000}{1.6 \times 10^{-19}} \text{ s}^{-1} = 6.25 \times 10^{21} \text{ s}^{-1}.$$

Let the number of protons that should strike the barrier for one proton to tunnel through the potential barrier be N. We thus have

$$NT_{proton} = 1,$$

and $N = 10^{132.8}$.

Therefore, the time required for N protons to strike the barrier will be

$$t = \frac{N}{n} = \frac{10^{132.8}}{6.25\times10^{21}}\ \text{s} = \frac{10^{132.8}}{6.25\times10^{21}}\times\frac{1}{3.156\times10^{7}}\ \text{years}$$

$$= 5.069\times10^{-2}\times10^{104.8}\ \text{years}$$

$$= 10^{0.7}\times10^{102.8}\ \text{years}$$

$$= 10^{103.5}\ \text{years}.$$

We have found that in order to see one proton to tunnel through the barrier we may have to wait for

$\approx 10^{104}$ years!

(b)

And, the transmission probability for an electron to tunnel through the barrier will therefore be

$$T_{electron} \approx \exp\left(-2\times 7.0\times 10^{-10} \times \sqrt{\frac{8\pi^2 \times 9.11\times 10^{-31} \times 1.0 \times 1.6\times 10^{-19}}{\left(6.63\times 10^{-34}\right)^2}} \right)$$

$$= \exp\left(-2\times 7.0\times 10^{-10} \times 5.11\times 10^{9}\right)$$

$$= \exp\left(-7.154\right) = 7.82\times 10^{-4}.$$

Let the number of electrons that should strike the barrier for one electron to tunnel through the potential barrier be N'. We thus have

$$N' T_{electron} = 1,$$

and

$$N' = \frac{1}{7.82\times 10^{-4}} = 1.28\times 10^{3}.$$

Therefore, the time required for N' electrons to strike the barrier will be

$$t' = \frac{N'}{n} = \frac{1.28\times 10^{3}}{6.25\times 10^{21}} \text{ s} = 2.05\times 10^{-19} \text{ s}.$$

We thus find that in order to see one electron to tunnel through the barrier we only have to wait for 2.05×10^{-19} s. We notice that the smaller mass of electron compared to that of proton makes an enormous difference in the tunnelling probability.