741.

Problem 39.79P (HRW)

A proton and a deuteron (*the latter has the same charge as a proton but twice the mass*) *strike a potential energy barrier that is* 10 fm *thick and* 10 MeV *high*. *Each particle has a kinetic energy of* 3.0 MeV *before it strikes the barrier*. *We have to find* (a) *the transmission probability for each*; (b) *their respective kinetic energies after they pass through the barrier* (*assuming that they do so*); *and* (c) *their respective kinetic energies if they are reflected from the barrier*.

Solution:

We will need the following data:

Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg,

Mass of a deuteron, $m_d = 2m_p = 3.34 \times 10^{-27}$ kg.

The transmission probability of a particle of mass *m*, energy *E* through a barrier of height *U* and width *L* is given approximately by the function

$$T \approx \exp\left(-2L\sqrt{\frac{8\pi^2 m(U-E)}{h^2}} \right).$$

Thickness of the barrier, $L = 10 \text{ fm} = 1.0\times10^{-14}\text{m}$,

height of the barrier, $U = 10$ MeV$=1.6\times10^{-12}$ J,

kinetic energy of the protons and kinetic energy of the deuterons before they strike the barrier are equal and each is $E = 3.0 \text{ MeV} = 4.8\times10^{-13}$ J.

(a)

We calculate next the transmission probabilities.

Transmission probability for a proton to tunnel through the barrier will be

$$T_{proton} \approx \exp\left(-2L\sqrt{\frac{8\pi^2 m_p(U-E)}{h^2}} \right)$$

$$= \exp\left(-2\times1.0\times10^{-14}\sqrt{\frac{8\pi^2\times1.67\times10^{-27}\times7\times1.6\times10^{-13}}{\left(6.63\times10^{-34}\right)^2}} \right)$$

$$= \exp\left(-2\times10^{-14}\times5.796\times10^{14}\right)$$

$$= \exp\left(-11.59\right) = 9.2\times10^{-6}.$$

And, the transmission probability for a deuteron to tunnel through the barrier will be

$$T_{deuteron} \approx \exp\left(-2L\sqrt{\frac{8\pi^2 m_d \left(U-E\right)}{h^2}} \right)$$

$$= \exp\left(-2\times 1.0\times 10^{-14} \sqrt{\frac{8\pi^2 \times 1.67\times 2\times 10^{-27} \times 7\times 1.6\times 10^{-13}}{\left(6.63\times 10^{-34}\right)^2}} \right)$$

$$= \exp\left(-2\times 10^{-14}\times 8.196\times 10^{14}\right)$$

$$= \exp\left(-16.39\right) = 7.6\times 10^{-8}.$$

(b)

After tunnelling through the barrier, protons and neutrons move through a zero potential region, the kinetic energy of each will therefore be 3.0 MeV.

(c)

On reflection from the barrier, protons and neutrons move through a zero potential region, the kinetic energy of each will therefore be 3.0 MeV.