740.

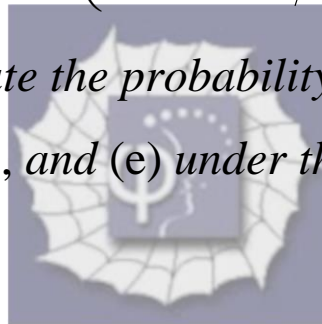## Problem 50.38 (RHK)

A particle is confined between rigid walls separated by a distance L. We have to show that (a) *the probability that it will be found within a distance $L/3$ from one wall is given by*

$$P = \frac{1}{3}\left(1 - \frac{\sin(2\pi n/3)}{2\pi n/3}\right).$$

We have to evaluate the probability for (b) $n=1$, (c) $n=2$, (d) $n=3$, *and* (e) *under the assumption of classical physics.*

Solution:

(a)

The wave functions of a particle that is trapped in an infinitely deep well of length L are

$$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right), \quad n=1,2,3...$$

Note that

$$\psi_n(0) = \psi_n(L) = 0.$$

The probability density will be

$$P_n(x) = A^2 \sin^2\left(\frac{n\pi x}{L}\right).$$

We normalise $P_n(x)$ by requiring that the probability of finding the particle any where between $x = 0$ and $x = L$ has to be one. That is we have the condition

$$\int_0^L P_n(x)\,dx = 1 \text{ , or}$$

$$A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) = A^2 \frac{L}{2} = 1,$$

or

$$A = \sqrt{\frac{2}{L}}.$$

The normalised probability density function in the state n is therefore

$$P_n(x) = \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right).$$

The probability of finding the particle within a distance $L/3$ from one wall will therefore be

$$P = \int_0^{L/3} P_n(x)\,dx = \frac{2}{L} \int_0^{L/3} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \int_0^{L/3} \left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right) dx$$

$$= \frac{1}{L}\left(\frac{L}{3} - \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{3}\right)\right)$$

$$= \frac{1}{3}\left(1 - \frac{\sin(2n\pi/3)}{2n\pi/3}\right).$$

(b)

The probability for $n=1$ will be

$$P = \frac{1}{3}\left(1 - \frac{\sin(2\pi/3)}{2\pi/3}\right) = 0.19.$$

(c)

The probability for $n=2$ will be

$$P = \frac{1}{3}\left(1 - \frac{\sin(4\pi/3)}{4\pi/3}\right) = 0.40.$$

(d)

The probability for $n=3$ will be

$$P = \frac{1}{3}\left(1 - \frac{\sin(6\pi/3)}{6\pi/3}\right) = 0.33.$$

(e)

Under the assumption of classical physics, the probability of finding a particle that is trapped between rigid walls separated by a length L will be uniform, and therefore the probability of finding the particle within a distance $L/3$ from one wall will be

$P = 0.33$.