738.

Problem 50.34 (RHK)

We consider a conduction electron in a cubical crystal of a conducting material. Such an electron is free to move throughout the volume of the crystal but cannot escape to the outside. It is trapped in a three-dimensional infinite well. The electron can move in three dimensions, so that its total energy is given by

$$E_{n_1,n_2,n_3} = \frac{h^2}{8mL^2}\left(n_1^2 + n_2^2 + n_3^2\right),$$

in which n_1, n_2, n_3 *each take on the values* 1, 2, ... *We have to calculate the energies of the lowest five distinct states for a conducting electron moving in a cubical crystal of edge length* $L = 250\ \text{nm}$.

Solution:

The total energy of an electron moving in a cubical crystal of edge length L is given by the equation

$$E_{n_1,n_2,n_3} = \frac{h^2}{8mL^2}\left(n_1^2 + n_2^2 + n_3^2\right),$$

in which n_1, n_2, n_3 each take on the values 1, 2, ...

Therefore, the lowest energy five distinct states will have the quantum numbers

$$n_1, n_2, n_3 = (1,1,1),\ (2,1,1),\ (1,2,1),\ (1,1,2),\ (2,2,1)$$

It is given that $L = 250\text{ nm}$.

We note that

$$\frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.11\times10^{-31}\times\left(250\times10^{-9}\right)^2}\ \text{J}$$

$$= 9.65\times10^{-25}\ \text{J}$$

$$= \frac{9.65\times10^{-25}}{1.6\times10^{-19}}\ \text{eV} = 6.0\ \mu\text{eV}.$$

Therefore, the energies of the lowest five distinct states will be

$$E_{1,1,1} = 18.0\ \mu\text{eV},$$

$$E_{2,1,1} = E_{1,2,1} = E_{1,1,2} = 36.0\ \mu\text{eV},$$

and

$$E_{2,2,1} = 54\ \mu\text{eV}.$$