737.

Problem 50.32 (RHK)

We have to calculate (a) *the fractional energy difference between two adjacent energy levels of a particle confined in a one-dimensional well of infinite depth*; (b) *and discuss the result in terms of the correspondence principle*.

Solution:

(a)

Let a particle of mass m be confined in a square well of width L and infinite depth. The energy levels of the particle are given by the equation

$$E_n = \frac{n^2 h^2}{8mL^2},\ n = 1, 2, 3, ..$$

The difference energy of levels with quantum numbers n and $n+1$ will be

$$\Delta E_n = E_{n+1} - E_n = \frac{h^2}{8mL^2}\left((n+1)^2 - n^2\right)$$

$$= \frac{h^2}{8mL^2}(2n+1).$$

Therefore, the fractional energy difference between two adjacent energy levels of a particle confined in a one-dimensional well of infinite depth will be

$$\frac{\Delta E_n}{E_n} = \frac{2}{n} + \frac{1}{n^2}.$$

(b)

The correspondence principle states that in the limit of large quantum numbers the quantum system behaves like a classical system. We calculate the fractional shift in energy levels having successive quantum numbers in the limit of large quantum number. That is we calculate

$$\lim_{n?\ 1} \frac{\Delta E_n}{E_n} ; \frac{2}{n}.$$

The classical energy of a particle of mass m and momentum p is

$$E = \frac{p^2}{2m}.$$

Therefore,

$$\frac{\Delta E}{E} = \frac{2\Delta p}{p}.$$

We recall that momentum of a particle in an infinite square well in state with quantum number n is

$$p_n = \frac{nh}{2L}.$$

Therefore,

$$\frac{\Delta p_n}{p_n} = \frac{1}{n}.$$

We thus note that

$$\lim_{n ?\ 1} \ \frac{\Delta E_n}{E_n} \ ; \ \frac{2}{n} = \frac{\Delta p_n}{p_n},$$

which is what we would expect for a free particle in classical mechanics. We have thus verified the correspondence principle.