736.

<u>Problem 50.29 (RHK)</u>

We have to calculate (a) *the smallest allowed energy of an electron confined to an infinitely deep well with a width equal to the diameter of an atomic nucleus* (*about* 1.4×10^{-14} m); (b) *repeat the calculation for a neutron*; (c) *compare these results with the binding energy* (*several* MeV) *of protons and neutrons inside the nucleus. On this basis, we have to answer whether electrons should be expected inside nuclei.*

<u>Solution:</u>

Let the width of the infinite square well potential be *L*. In an infinite square well the wave function has to vanish at $x=0$ and $x=L$. Therefore, the wave function for the ground state as it will correspond to the longest allowed wavelength will be

$$\psi_1 = A\sin\left(\frac{\pi x}{L}\right) = A\sin kx\ .$$

The wave number *k* for this wave function will be

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{L},$$

and therefore the de Broglie wavelength of a particle in this state will be

$$\lambda = 2L.$$

Using de Broglie equation, we note that the momentum of a particle of mass *m* in the lowest energy state of an infinite square well of width *L* will be

$$p = \frac{h}{2L},$$

and the energy of the lowest energy state will be

$$E_1 = \frac{p^2}{2m} = \frac{h^2}{8mL^2}.$$

In the model of the nucleus suggested in the problem, the width of the square well

$$L = 1.4\times10^{-14}\ \text{m}.$$

(a)

Mass of an electron $m_e = 9.11\times10^{-31}$ kg, therefore, the ground state energy of electron in this model will be

$$(E_1)_{electron} = \frac{h^2}{8m_eL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.11\times10^{-31}\times\left(1.4\times10^{-14}\right)^2}\ \text{J}$$

$$= 0.31\times10^{-9}\ \text{J}$$

$$= \frac{0.31\times10^{-9}}{1.6\times10^{-13}}\ \text{MeV} = 1923\ \text{MeV.}$$

(b)

Mass of a neutron $m_n = 1.67\times10^{-27}\ \text{kg}$, therefore, the ground state energy of neutron in this model will be

$$(E_1)_{neutron} = \frac{h^2}{8m_nL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times1.67\times10^{-27}\times\left(1.4\times10^{-14}\right)^2}\ \text{J}$$

$$= 1.68\times10^{-13}\ \text{J}$$

$$= \frac{1.68\times10^{-13}}{1.6\times10^{-13}}\ \text{MeV} = 1.05\ \text{MeV.}$$

(c)

As the binding energy of neutron and proton inside a nucleus is of the order of MeV, and the energy of neutron as worked out in the infinite square well model is also of the same order it is reasonable to assume that neutrons and protons are confined in a size of the order of $1.4\times10^{-14}\ \text{m}$. Te energy of an electron in this model is of the order of 2000 MeV, which is much greater than the nuclear energies. Therefore, electrons cannot be confined inside a nucleus.