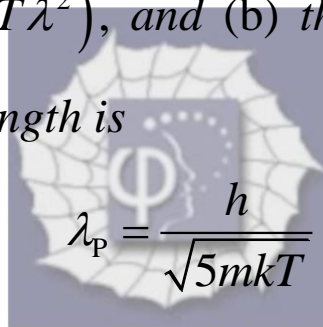


732.

Problem 50.20 (RHK)

A beam of atoms emerges from an oven that is at a temperature T . The distribution of the speeds of the atoms in the beam is proportional to $v^3 \exp(-mv^2/2kT)$.

(a) We have to show that the distribution of the de Broglie wavelengths of the atoms is proportional to $\lambda^{-5} \exp(-h^2/2mkT\lambda^2)$, and (b) that the most probable de Broglie wavelength is


$$\lambda_p = \frac{h}{\sqrt{5mkT}}.$$

Solution:

(a)

The distribution of the speeds of the atoms in the beam is proportional to $v^3 \exp(-mv^2/2kT)$. That is the number of atoms in the beam having speed between v and $v + dv$ is given by the equation

$$N(v)dv = \text{const. } v^3 e^{-mv^2/2kT} dv.$$

The relation between speed v of atoms and their de Broglie wavelength is

$$\lambda = \frac{h}{mv},$$

where m is the mass of the atom. We thus have the relation

$$|dv| = \frac{h}{m\lambda^2} d\lambda.$$

We define a function $N(\lambda)$ by the relation

$$N(\lambda)d\lambda = N(v)dv.$$

Therefore,

$$N(\lambda)d\lambda = N(v)dv = \text{const. } v^3 e^{-mv^2/2kT} \left(\frac{h}{m}\right) \frac{d\lambda}{\lambda^2},$$

or

$$N(\lambda) = \text{const. } \frac{1}{\lambda^5} \left(\frac{h}{m}\right)^4 e^{-h^2/2mkT\lambda^2}.$$

That is

$$N(\lambda) \propto \frac{1}{\lambda^5} e^{-h^2/2mkT\lambda^2}.$$

(b)

We find the most probable value of $N(\lambda)$ by requiring the condition

$$\left(\frac{dN(\lambda)}{d\lambda} \right)_{\lambda=\lambda_p} = 0.$$

This gives

$$\left(-\frac{5}{\lambda_p^6} + \frac{1}{\lambda_p^5} \left(\frac{h^2}{mkT \lambda_p^3} \right) \right) \exp\left(-h^2/2mkT \lambda_p^2\right) = 0,$$

or

$$\lambda_p^2 = \frac{h^2}{5mkT},$$

or

$$\lambda_p = \frac{h}{\sqrt{5mkT}}.$$

