**732.**

**Problem 50.20 (RHK)**

*A beam of atoms emerges from an oven that is at a temperature T. The distribution of the speeds of the atoms in the beam is proportional to* $v^3 \exp\left(-mv^2/2kT\right)$.

(a) *We have to show that the distribution of the de Broglie wavelengths of the atoms is proportional to* $\lambda^{-5} \exp\left(-h^2/2mkT\lambda^2\right)$, *and* (b) *that the most probable de Broglie wavelength is*

$$\lambda_{\mathrm{P}} = \frac{h}{\sqrt{5mkT}} .$$

**Solution:**

(a)

The distribution of the speeds of the atoms in the beam is proportional to $v^3 \exp\left(-mv^2/2kT\right)$. That is the number of atoms in the beam having speed between $v$ and $v+dv$ is given by the equation

$$N(v)\,dv = const.\; v^3 e^{-mv^2/2kT} dv.$$

The relation between speed $v$ of atoms and their de Broglie wavelength is

$$\lambda = \frac{h}{mv},$$

where $m$ is the mass of the atom. We thus have the relation

$$\left|dv\right| = \frac{h}{m\lambda^2} d\lambda.$$

We define a function $N(\lambda)$ by the relation

$$N(\lambda)d\lambda = N(v)dv.$$

Therefore,

$$N(\lambda)d\lambda = N(v)dv = const.\, v^3 e^{-mv^2/2kT} \left(\frac{h}{m}\right)\frac{d\lambda}{\lambda^2},$$

or

$$N(\lambda) = const.\frac{1}{\lambda^5}\left(\frac{h}{m}\right)^4 e^{-h^2/2mkT\lambda^2}.$$

That is

$$N(\lambda) \propto \frac{1}{\lambda^5} e^{-h^2/2mkT\lambda^2}.$$

(b)

We find the most probable value of $N(\lambda)$ by requiring the condition

$$\left( \frac{dN(\lambda)}{d\lambda} \right)_{\lambda=\lambda_P} = 0.$$

This gives

$$\left( -\frac{5}{\lambda_P^6} + \frac{1}{\lambda_P^5} \left( \frac{h^2}{mkT\lambda_P^3} \right) \right) \exp\left( -h^2 / 2mkT\lambda_P^2 \right) = 0,$$

or

$$\lambda_P^2 = \frac{h^2}{5mkT},$$

or

$$\lambda_P = \frac{h}{\sqrt{5mkT}}.$$