

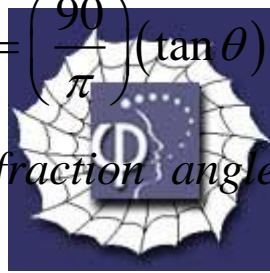
731.

Problem 50.19 (RHK)

A beam of low-energy neutrons emerges from a reactor and is diffracted from a crystal. The kinetic energy of the neutrons is contained in a band of width ΔK centred on the kinetic energy K . We have to show that the angles for a given order of diffraction are spread over a range $\Delta\theta$ given in degrees by

$$\Delta\theta = \left(\frac{90}{\pi}\right)(\tan\theta)\frac{\Delta K}{K},$$

Where θ is the diffraction angle for a neutron with kinetic energy K .



Solution:

The de Broglie wavelength of a neutron of kinetic energy K is given by the expression

$$\lambda = \frac{h}{\sqrt{2m_{\text{neutron}}K}},$$

as momentum of a neutron of mass m_{neutron} is

$$p = \sqrt{2m_{\text{neutron}}K}.$$

Therefore, as the spread of kinetic energy is ΔK centred on the kinetic energy K the corresponding spread of de Broglie wavelengths will be

$$\Delta\lambda = \frac{1}{2} \frac{\lambda}{K} \Delta K.$$

This is obtained by differentiating the de Broglie wavelength with respect to the variable K .

If the grating separation is d , the grating equation for diffraction maximum of order m at diffraction angle θ will be

$$d \sin \theta = m\lambda.$$

Therefore, the spread of diffraction angle if the spread of wavelength is $\Delta\lambda$ will



$$\Delta\theta = \frac{m\Delta\lambda}{d \cos \theta} \text{ rad,} \quad \text{be}$$

or

$$\begin{aligned} \Delta\theta &= \frac{m}{d \cos \theta} \left(\frac{1}{2} \frac{\lambda}{K} \right) \Delta K \text{ rad,} \\ &= \frac{1}{2} \frac{\tan \theta}{K} \Delta K \text{ rad} \\ &= \left(\frac{180}{\pi} \right) \frac{1}{2} \frac{\tan \theta}{K} \Delta K \text{ degrees} \\ &= \left(\frac{90}{\pi} \right) (\tan \theta) \frac{\Delta K}{K} \text{ degrees.} \end{aligned}$$