

728.

Problem 50.8 (RHK)

The highest achievable resolving power is limited only by the wavelength used; that is, the smallest detail that can be separated is about equal to the wavelength. Suppose we wish to “see” inside an atom. Assuming the atom to have a diameter of 100 pm, that we wish to resolve detail of separation about 10 pm. (a) If an electron microscope is used, we have to find the minimum energy of the electron. (b) If a light microscope is used, we have to find the minimum energy of the photons that is needed. (c) We have to select of the two microscopes the one that is suitable for the purpose.

Solution:

(a)

The momentum of an electron with de Broglie wavelength of 10 pm will be

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10 \times 10^{-12}} \text{ kg m s}^{-1}$$
$$= 6.63 \times 10^{-23} \text{ kg m s}^{-1}.$$

The speed of the electron will be

$$v = \frac{p}{m} = \frac{6.63 \times 10^{-23}}{9.11 \times 10^{-31}} \text{ m s}^{-1} = 0.727 \times 10^8 \text{ m s}^{-1} \\ = 0.24 c.$$

We calculate the value of pc .

We find

$$pc = 6.63 \times 10^{-23} \times 3 \times 10^8 \text{ J} \\ = 1.989 \times 10^{-14} \text{ J} = 1.989 \times 10^{-14} \times \frac{1}{1.6 \times 10^{-13}} \text{ MeV} \\ = 0.124 \text{ MeV}.$$

The kinetic energy of the electron with momentum

$p = 6.63 \times 10^{-23} \text{ kg m s}^{-1}$ will therefore be

$$KE = \frac{(6.63 \times 10^{-23})^2}{2 \times 9.11 \times 10^{-31}} \text{ J} \\ = 2.41 \times 10^{-15} \text{ J} = \frac{2.41 \times 10^{-15}}{1.6 \times 10^{-19}} \text{ eV} \\ = 1.51 \times 10^4 \text{ eV} = 15.1 \text{ keV}.$$

(b)

The energy of a photon of wavelength 10 pm will be

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10 \times 10^{-12}} \text{ J} \\ = 0.124 \text{ MeV}.$$

(c)

As the photon energy is in the gamma ray region, photons of such energy can be obtained only in nuclear transitions and a high intensity gamma ray beam will be difficult to obtain.

On the other hand, it will be easy to have an electron beam of kinetic energy 15.1 keV, which can be obtained by accelerating electrons across 15.1 kV potential.

Therefore, for probing an atom an electron microscope is to be preferred to a light microscope.

