726.

Problem 50.2 (RHK)

Using the classical relation between momentum and kinetic energy, we have to show that the de Broglie wavelength of an electron can be written (a) *as*

$$\lambda = \frac{1.227 \text{ nm}}{\sqrt{K}},$$

in which K is the kinetic energy in electron volts, or (b) *as*

$$\lambda = \sqrt{\frac{1.50}{V}},$$

where λ is in nm, and V is the accelerating potential in volts.

Solution:

In answering this problem we will use the values of the fundamental constants as given below:

Electron rest mass, $m_e = 9.11 \times 10^{-31}$ kg,

Elementary charge, $e = 1.60 \times 10^{-19}$ C ,

Planck constant, $h = 6.626 \times 10^{-34}$ J s.

(a)

Momentum of a nonrelativistic electron having kinetic energy K J is given by the expression

$$p = \sqrt{2mK}\ .$$

De Broglie wavelength of a particle of momentum p is given by the relation

$$\lambda = \frac{h}{p}.$$

We note in the mks system of units energy is to be expressed in joules. As the kinetic energy of the electron is given in eV, we write its equivalent in joules.

$$K\ \text{eV} = 1.6\times10^{-19} K\ \text{J},$$

as

$$1\ \text{eV} = 1.6\times10^{-19}\ \text{J}.$$

Therefore, de Broglie wavelength of an electron of kinetic energy K eV will be given by the expression

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\ \text{J s}}{\sqrt{2\times9.11\times10^{-31}\ \text{kg}\times1.6\times10^{-19} K\ \text{J}}}$$

$$= \frac{1.227\times10^{-9}}{\sqrt{K}}\ \text{m} = \frac{1.227}{\sqrt{K}}\ \text{nm}.$$

(b)

Kinetic energy of an electron that has been accelerated across a potential difference of V volt will be

$K = 1.6 \times 10^{-19} V$ J. Its de Broglie wavelength will be

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J s}}{\sqrt{2 \times 9.11 \times 10^{-31} \times \text{kg} \times 1.6 \times 10^{-19} \times V \text{ C V}}}$$

$$= \frac{1.227}{\sqrt{V}} \text{ nm} = \sqrt{\frac{1.5}{V}} \text{ nm.}$$