724.

Problem 39.41P (HRW)

An electron of mass m and speed v undergoes a head-on collision with a gamma-ray photon of energy hf_0, *scattering the gamma-ray photon back in the direction of incidence*. We have to verify that the energy of the scattered *gamma-ray photon, as measured in the laboratory system, is*

$$E' = hf' = hf_0\left(1+\frac{2hf_0}{mc^2}\sqrt{\frac{1+v/c}{1-v/c}}\right)^{-1} .$$

Solution:

In answering this problem we will use relativistic mechanics and the property that a photon of frequency f has momentum hf/c in the direction of motion of the photon.

Let the speed of the electron that is moving toward the photon, in the laboratory frame, be v, and the frequency of the incident photon be f_0. Let us assume that photon and electron are back scattered. Let the speed of the electron after the head-on collision be v' and that the frequency of the back scattered photon be f'.

We will write the relativistic equations for energy and momentum conservation.

Energy conservation equation is

$$hf_0 + \frac{mc^2}{\sqrt{1 - v^2/c^2}} = hf' + \frac{mc^2}{\sqrt{1 - v'^2/c^2}} \ . \ (1)$$

Momentum conservation equation in head-on collision of a photon and electron when they are back scattered is

$$\frac{hf_0}{c} - \frac{mv}{\sqrt{1 - v^2/c^2}} = -\frac{hf'}{c} + \frac{mv'}{\sqrt{1 - v'^2/c^2}} \ . \ (2)$$

We have two linear equations in two unknowns f' and v'. We solve these equations algebraically. We find that

$$\frac{mc^2}{\sqrt{1 - v'^2/c^2}} = \frac{\left\{ \frac{1}{2} mc^2 \left(1 + \left(\frac{2hf_0}{mc^2} + \frac{\sqrt{(1 - v/c)}}{\sqrt{(1 + v/c)}} \right)^2 \right) \right\}}{\frac{2hf_0}{mc^2} + \frac{\sqrt{(1 - v/c)}}{\sqrt{(1 + v/c)}}}.$$

The expression for the energy of the gamma-ray photon that is back scattered can be found from the equation for energy conservation. We have

$$hf' = hf_0 + \frac{mc^2}{\sqrt{1 - v^2/c^2}} - \frac{mc^2}{\sqrt{1 - v'^2/c^2}}.$$

We now substitute the expression for $\frac{mc^2}{\sqrt{1 - v'^2/c^2}}$ and carry out algebraic simplifications. We find that

$$E' = hf' = hf_0 \left(1 + \frac{2hf_0}{mc^2} \sqrt{\frac{1 + v/c}{1 - v/c}} \right)^{-1} .$$