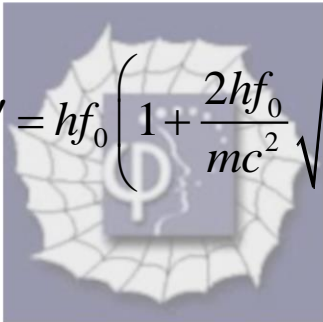


724.

Problem 39.41P (HRW)

An electron of mass m and speed v undergoes a head-on collision with a gamma-ray photon of energy hf_0 , scattering the gamma-ray photon back in the direction of incidence. We have to verify that the energy of the scattered gamma-ray photon, as measured in the laboratory system, is


$$E' = hf' = hf_0 \left(1 + \frac{2hf_0}{mc^2} \sqrt{\frac{1+v/c}{1-v/c}} \right)^{-1} .$$

Solution:

In answering this problem we will use relativistic mechanics and the property that a photon of frequency f has momentum hf/c in the direction of motion of the photon.

Let the speed of the electron that is moving toward the photon, in the laboratory frame, be v , and the frequency of the incident photon be f_0 . Let us assume that photon and electron are back scattered. Let the speed of the

electron after the head-on collision be v' and that the frequency of the back scattered photon be f' .

We will write the relativistic equations for energy and momentum conservation.

Energy conservation equation is

$$hf_0 + \frac{mc^2}{\sqrt{1-v^2/c^2}} = hf' + \frac{mc^2}{\sqrt{1-v'^2/c^2}} \quad (1)$$

Momentum conservation equation in head-on collision of a photon and electron when they are back scattered is

$$\frac{hf_0}{c} - \frac{mv}{\sqrt{1-v^2/c^2}} = -\frac{hf'}{c} + \frac{mv'}{\sqrt{1-v'^2/c^2}} \quad (2)$$

We have two linear equations in two unknowns f' and v' . We solve these equations algebraically. We find that

$$\frac{mc^2}{\sqrt{1-v'^2/c^2}} = \frac{\left\{ \frac{1}{2} mc^2 \left(1 + \left(\frac{2hf_0}{mc^2} + \frac{\sqrt{(1-v/c)}}{\sqrt{(1+v/c)}} \right)^2 \right) \right\}}{\frac{2hf_0}{mc^2} + \frac{\sqrt{(1-v/c)}}{\sqrt{(1+v/c)}}}.$$

The expression for the energy of the gamma-ray photon that is back scattered can be found from the equation for energy conservation. We have

$$hf' = hf_0 + \frac{mc^2}{\sqrt{1-v^2/c^2}} - \frac{mc^2}{\sqrt{1-v'^2/c^2}}.$$

We now substitute the expression for $\frac{mc^2}{\sqrt{1-v'^2/c^2}}$ and

carry out algebraic simplifications. We find that

$$E' = hf' = hf_0 \left(1 + \frac{2hf_0}{mc^2} \sqrt{\frac{1+v/c}{1-v/c}} \right)^{-1}.$$

