721.

Problem 49.57 (RHK)

We have to show that $\Delta E/E$, the fractional loss of energy of a photon during Compton collision is given by

$$\frac{\Delta E}{E} = \frac{h\nu'}{mc^2}(1-\cos\phi).$$

Solution:

Let the frequencies of the incident and scattered photon be ν, and ν', respectively.

The loss of energy of the photon in Compton scattering will be given by

$$\Delta E = E - E' = h(\nu - \nu') = h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) = hc\frac{(\lambda' - \lambda)}{\lambda\lambda'},$$

where λ, and λ' are the wavelengths of the incident and scattered photons, respectively.

Substituting,

$$\frac{hc}{\lambda} = E,$$

we get

$$\frac{\Delta E}{E} = \frac{(\lambda' - \lambda)}{\lambda'}.$$

Using the Compton scattering formula

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi),$$

We rewrite

$$\frac{\Delta E}{E} = \frac{(\lambda' - \lambda)}{\lambda'} = \frac{h}{mc\lambda'}(1 - \cos\phi)$$

$$= \frac{E'}{mc^2}(1 - \cos\phi).$$