713.

Problem 49.26 (RHK)

We may assume that 25.0 g *of aluminium at* 80.0 K *are mixed thoroughly with* 12.0 g *of aluminium at* 200 K *in an insulated container. We have to find the final temperature of the mixture. We may assume that Einstein's theory of heat capacities is valid and that, at these relative low temperatures, the differences between the heat capacity at constant volume and that at constant pressure may be neglected. We may assume further that there are no energy exchanges between the two aluminium specimens and the container. The Einstein temperature of aluminium may be taken to be* 290 K.

Solution:

Molar mass of aluminium is 26.98 g.

Einstein temperature for aluminium, $T_E = 290\ \text{K}$.

According to Einstein's theory of heat capacity the internal energy of one mole at temperature is given by the function

$$E_{\text{int}} = 3RT_E\left(\frac{1}{e^x - 1}\right),$$

$$x = \frac{T_E}{T}.$$

Therefore, total internal energy of 25.0 g of aluminium at 80.0 K and 12.0 g of aluminium at 200 K in an insulated container will be

$$E = 3RT_E \times \frac{1}{26.98} \times \left(25.0 \times \frac{1}{e^{290/80} - 1} + 12 \times \frac{1}{e^{290/200} - 1}\right)$$

$$= 3RT_E \times \frac{1}{26.98} \times (0.684 + 3.677)$$

$$= 3RT_E \times \frac{1}{26.98} \times 4.361.$$

Assuming that when the two pieces of aluminium are intimately mixed without exchange of energy with the container, and further assuming that the energy at equilibrium can be determined by the Einstein's theory of heat capacity, and let the final common temperature of the total amount of aluminium be T K, from the conservation of energy, we write the equation

$$3RT_E \times \frac{1}{26.98} \times \left(37 \times \frac{1}{e^x - 1} \right) = 3RT_E \times \frac{1}{26.98} \times 4.361,$$

or

$$e^x - 1 = \frac{37}{4.361}$$
$$= 8.483$$

or

$$e^x = 9.483,$$

and

$$x = \frac{290 \text{ K}}{T} = 2.249, \text{ and } T = 129 \text{ K}.$$