712.

Problem 49.25 (RHK)

A 12.0-g *block of aluminium is heated from* 80 K *up to* 180 K, *under constant volume conditions*. *We have to find the amount of heat required according to* (a) *the classical theory of heat capacity and* (b) *Einstein's quantum theory of heat capacity*. *The Einstein temperature for aluminium may be taken to be* 290 K.

Solution:

Molar mass of aluminium is 26.98 g. Therefore, the number of moles of aluminium atoms in 12.0 g of aluminium will be

$$n = \frac{12.0}{26.98}\ \text{mol} = 0.445\ \text{mol}.$$

(a)

According to the classical kinetic theory the internal energy per mole at temperature T is given by $3RT$. Therefore, change in internal energy of 12.0 g of aluminium when it is heated from 80 K to 180 K at constant volume will be

$$\left(\Delta E_{\text{int}}\right)_{classical} = 3R\left(T_f - T_i\right)n$$
$$= 3\times 8.31\times\left(180-80\right)\times 0.445 \text{ J}$$
$$= 1{,}109 \text{ J}.$$

(b)

According to Einstein's theory of heat capacity the molar internal energy is given by the function

$$E_{\text{int}} = 3RT_E\left(\frac{1}{e^x - 1}\right),$$

$x = T_E/T,$ where T_E is the *Einstein temperature.*

Therefore, change in internal energy of 12.0 g of aluminium when it is heated from 80 K to 180 K at constant volume will be

$$\left(\Delta E\right)_{Einstien} = 3RnT_E\left(\frac{1}{\exp\left(T_E/T_f\right)-1} - \frac{1}{\exp\left(T_E/T_i\right)-1}\right)$$
$$= 3\times 8.31\times 0.445\times 290\times\left(\frac{1}{e^{290/180}-1} - \frac{1}{e^{290/80}-1}\right) \text{J}$$
$$= 714 \text{ J}.$$