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Problem 49.21 (RHK)

In terms of Einstein's theory of heat capacity, we have to find (a) the molar heat capacity at constant volume of a solid at its Einstein temperature, and express our answer as a percentage of its classical value of $3R$. (b) We have to find the molar internal energy at the Einstein temperature, and express our answer as a percentage of its classical value of $3RT_E$.



Solution:

The molar internal energy E_{int} of a solid, according to Einstein's theory of heat capacity, can be expressed as

$$E_{\text{int}} = 3RT_E \left(\frac{1}{e^x - 1} \right),$$

in which $x = T_E/T$, where $T_E = hv/k$ is the *Einstein temperature*.

Specific heat at constant volume

$$C_V = \frac{dE_{\text{int}}}{dT} = \frac{d}{dT} \left(3RT_E \left(\frac{1}{e^x - 1} \right) \right) \\ = 3RT_E \left(\frac{1}{e^x - 1} \right)^2 \times \frac{T_E}{T^2}.$$

Therefore, the molar heat capacity at constant volume of a solid at its Einstein temperature will be

$$C_V (T = T_E) = 3R \times \frac{1}{(e - 1)^2} = 0.338 \times (3R).$$

It is 33.8% of its classical value $3R$.

(b)

The molar internal energy at the *Einstein temperature* will be

$$E_{\text{int}} (T = T_E) = 3RT_E \times \frac{1}{e - 1} = 0.582 \times (3RT_E),$$

which is 58.2% of its classical value $3RT_E$.