

708.

Problem 49.14 (RHK)

The filament of a particular 100-W light bulb is cylindrical wire of tungsten 0.280 mm in diameter and 1.80 cm long. Assuming an emissivity of unity and ignoring absorption of energy by the filament from the surroundings, we have to calculate (a) the operating temperature of the filament; (b) the time it will take filament to cool by 500 C^0 after the bulb is switched off.



Solution:

(a)

We will determine first the operating temperature of the filament. The surface area of the filament will be πdl , where d is the diameter and l is the length of the filament.

As $d = 0.280\text{ mm} = 2.8 \times 10^{-4}\text{ m}$, and

$l = 1.80\text{ cm} = 1.8 \times 10^{-2}\text{ m}$,

the surface area of the filament is

$$a = \pi \times 2.8 \times 10^{-4} \times 1.8 \times 10^{-2}\text{ m}^2.$$

Let the temperature of the filament of the light bulb when it is radiating at 100 W be T K. Using the Stefan-Boltzmann law we write the equation from which temperature of the filament can be determined.

$$\sigma T^4 a = 100 \text{ W},$$

or

$$5.67 \times 10^{-8} \times 1.58 \times 10^{-5} T^4 \text{ W} = 100 \text{ W},$$

or

$$T^4 = \frac{10^2}{5.67 \times 1.58 \times 10^{-13}} \text{ K}^4 = 111.6 \times 10^{12} \text{ K}^4,$$

or

$$T = (111.6)^{1/4} \times 10^3 \text{ K} = 3250 \text{ K}.$$

(b)



For answering the second part of the problem, we will use the following physical properties of tungsten:

Density, $\rho = 19.3 \times 10^3 \text{ kg m}^{-3}$,

Specific heat, $C = 134 \text{ J kg}^{-1} \text{ K}^{-1}$.

When the bulb has been switched off and the temperature of the filament is T K, let the temperature of the filament drop by ΔT K in Δt s. The change in the heat energy of the filament will be

$$\Delta Q = -\rho \times (\pi r^2 l) \times \Delta T.$$

Using the Stefan-Boltzmann law, we write

$$\Delta Q = -\rho \times (\pi r^2 l) \times C \times \Delta T.$$

We thus obtain the following differential equation giving change in temperature of the filament with time.

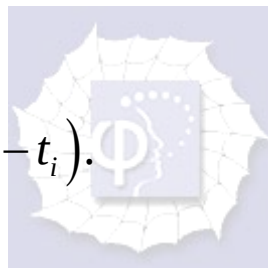
$$\frac{\Delta T}{T^4} = -\frac{2\sigma}{rC\rho} \Delta t.$$

We integrate this equation,

$$\int_{T_i}^{T_f} \frac{dT}{T^4} = -\frac{2\sigma}{rC\rho} (t_f - t_i),$$

or

$$\left(\frac{1}{T_f^3} - \frac{1}{T_i^3} \right) = \frac{6\sigma}{rC\rho} (t_f - t_i).$$



From the above equation we can calculate the time taken by the filament of the bulb to cool down by 500 C^0 from when the temperature of the filament was 3250 K . We have the data

$$T_i = 3250\text{ K},$$

and

$$T_f = 2750\text{ K}.$$

Therefore,

$$\begin{aligned}t_f - t_i &= \frac{rC\rho}{6\sigma} \left(\frac{1}{T_f^3} - \frac{1}{T_i^3} \right) \\&= \frac{0.140 \times 10^{-3} \times 134 \times 19.3 \times 10^3}{6 \times 5.67 \times 10^{-8}} \left(\frac{1}{2750^3} - \frac{1}{3250^3} \right) \text{ s} \\&= 2.0 \times 10^{-2} \text{ s} = 20 \text{ ms.}\end{aligned}$$

