708.

<u>Problem 49.14 (RHK)</u>

The filament of a particular 100-W *light bulb is cylindrical wire of tungsten* 0.280 mm *in diameter and* 1.80 cm *long*. *Assuming an emissivity of unity and ignoring absorption of energy by the filament from the surroundings, we have to calculate* (a) *the operating temperature of the filament*; (b) *the time it will take filament to cool by* $500\ \mathrm{C}^0$ *after the bulb is switched off.*

<u>Solution:</u>

(a)

We will determine first the operating temperature of the filament. The surface area of the filament will be πdl, where d is the diameter and l is the length of the filament.

As $d = 0.280\ \mathrm{mm} = 2.8 \times 10^{-4}\ \mathrm{m}$, and

$l = 1.80\ \mathrm{cm} = 1.8 \times 10^{-2}\ \mathrm{m}$,

the surface area of the filament is

$a = \pi \times 2.8 \times 10^{-4} \times 1.8 \times 10^{-2}\ \mathrm{m}^2$.

Let the temperature of the filament of the light bulb when it is radiating at 100 W be T K. Using the Stefan-Boltzmann law we write the equation from which temperature of the filament can be determined.

$$\sigma T^4 a = 100 \text{ W},$$

or

$$5.67\times10^{-8}\times1.58\times10^{-5}\,T^4 \text{ W} = 100 \text{ W},$$

or

$$T^4 = \frac{10^2}{5.67\times1.58\times10^{-13}} \text{ K}^4 = 111.6\times10^{12} \text{ K}^4,$$

or

$$T = (111.6)^{1/4}\times10^3 \text{ K} = 3250 \text{ K}.$$

(b)

For answering the second part of the problem, we will use the following physical properties of tungsten:

Density, $\rho = 19.3\times10^3$ kg m^{-3},

Specific heat, $C = 134$ J kg^{-1} K^{-1}.

When the bulb has been switched off and the temperature of the filament is T K, let the temperature of the filament drop by ΔT K in Δt s. The change in the heat energy of the filament will be

$$\Delta Q = -\rho\times\left(\pi r^2 l\right)\times\Delta T.$$

Using the Stefan-Boltzmann law, we write

$$\Delta Q = -\rho \times \left(\pi r^2 l\right) \times C \times \Delta T.$$

We thus obtain the following differential equation giving change in temperature of the filament with time.

$$\frac{\Delta T}{T^4} = -\frac{2\sigma}{rC\rho}\Delta t.$$

We integrate this equation,

$$\int_{T_i}^{T_f} \frac{dT}{T^4} = -\frac{2\sigma}{rC\rho}\left(t_f - t_i\right),$$

or

$$\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right) = \frac{6\sigma}{rC\rho}\left(t_f - t_i\right).$$

From the above equation we can calculate the time taken by the filament of the bulb to cool down by $500\ \mathrm{C}^0$ from when the temperature of the filament was 3250 K. We have the data

$T_i = 3250$ K,

and

$T_f = 2750$ K.

Therefore,

$$t_f - t_i = \frac{rC\rho}{6\sigma}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$$

$$= \frac{0.140\times10^{-3}\times134\times19.3\times10^{3}}{6\times5.67\times10^{-8}}\left(\frac{1}{2750^3} - \frac{1}{3250^3}\right)\ \text{s}$$

$$= 2.0\times10^{-2}\ \text{s} = 20\ \text{ms}.$$