

706.

Problem 49.12 (RHK)

A thermograph is a medical instrument used to measure radiation from the skin. For example, normal skin radiates at a temperature of 34°C and the skin over a tumour radiates at a slightly higher temperature.

(a) We have to derive an approximate expression for the fractional difference $\Delta I/I$ in the radiant intensity between adjacent areas of the skin that are at slightly different temperatures T and $T + \Delta T$. (b) We have to evaluate this expression for a temperature difference of 1.3 C° . We may assume that skin radiates with a constant emissivity.

Solution:

(a)

We use the Stefan-Boltzmann law for the radiant intensity $I(T)$:

$$I(T) = \varepsilon\sigma T^4,$$

where ε is the emissivity and the Stefan constant

$$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}.$$

Therefore,

$$\Delta I = 4\varepsilon\sigma T^3,$$

and

$$\frac{\Delta I}{I} = \frac{4\Delta T}{T}.$$

(b)

We next work out the expression $\Delta I/I$ for

$$T = (273 + 34 = 307) \text{ K and } \Delta T = 1.3.$$

We find

$$\frac{\Delta I}{I} = \frac{4 \times 1.3}{307} = 0.017 \text{ or } 1.7\%.$$

