701.

Problem 48.23 (RHK)

We may assume that a parallel of circularly polarized light whose power is 106 W *is absorbed by an object.* (a) *We have to find the rate at which angular momentum is transferred to the object.* (b) *Assuming that the object is a flat disk of diameter* 5.20 mm *and mass* 9.45 mg, *we have to find the time required for the disk to attain an angular speed of* $1.50\ \text{rev s}^{-1}$ (*we may assume that the disk is free to rotate about its axis*). *Let the wavelength of the incident light be* 516 nm.

Solution

Let ν be the frequency of incident light. According to light quantum hypothesis, each photon carries energy $h\nu$, where h is the Planck constant.

The object on which circularly polarized light is incident is a flat disk of diameter 5.20 mm. The power of the beam is 106 W. Therefore, the number of photons that are incident on the disk per second will be

$$N = \frac{106}{h\nu} \text{ photons s}^{-1}.$$

We will assume that all incident photons are absorbed by the disk. We will use the quantum property that each photon carries angular momentum of magnitude

$$l = \frac{h}{2\pi}.$$

Therefore, the total amount of angular momentum transferred to the disk per second by the photons will be

$$\mathrm{L} = Nl = \frac{106}{h\nu} \times \frac{h}{2\pi},$$

$$= \frac{106}{2\pi \times (c/\lambda)} \text{ kg m}^2 \text{ s}^{-2}$$

$$= \frac{106}{2\pi \times \left(3\times10^8 / 516\times10^{-9}\right)} \text{ kg m}^2 \text{ s}^{-2}$$

$$= 2.9\times10^{-14} \text{ kg m}^2 \text{ s}^{-2}.$$

Rotational inertia of a circular disk of radius R and mass M is $I = MR^2/2$. When the disk attains angular speed of 1.50 rev s^{-1}, its angular momentum will be

$$L_{disk} = \frac{M}{2} \times \frac{d^2}{4} \times 1.50 \times 2 \times \pi$$

$$= \frac{3.0 \times \pi \times 9.45 \times 10^{-6} \times \left(5.20 \times 10^{-3}\right)^2}{8} \text{ kg m}^2\text{s}^{-1}$$

$$= 3.01 \times 10^{-10} \text{kg m}^2\text{s}^{-1}$$

Therefore, the time in which the disk will attain rotational speed of 1.50 rev s^{-1} will be

$$t = \frac{3.01 \times 10^{-10}}{2.9 \times 10^{-14}} \text{ s} = 103.8 \times 10^2 \text{ s} = 2.88 \text{ hour.}$$