696.

Problem 48.12 (RHK)

It is desired to rotate the plane of vibration of a beam of polarized light by 90^0. (a) *We have to answer how it can be achieved using polarizing sheets*. (b) *We have to find the minimum number of sheets required in order for the total intensity loss to be less than* 5.0%.

Solution:

(a)

When the successive polarizing sheet is rotated by an angle θ with respect to the preceding sheet, the pane of vibration is rotated by the angle θ. So if n polarizing sheets are stacked with their characteristic angles rotated by angle θ with respect to the preceding sheet, the plane of vibration will get rotated by angle $n\theta$. Therefore, the plane of vibration of polarized light can be rotated by 90^0 by stacking n polarizing sheets such that

$$n\theta = \frac{\pi}{2}.$$

(b)

Intensity of light changes after each such rotation of the plane of vibration by a factor of $\cos^2\theta$. Let I_0 be the intensity of the incident polarized light. The intensity after passing through n polarizing sheets will become

$$I_n = I_0 \cos^{2n}\theta.$$

We want to find the minimum number of sheets required in order for the total intensity loss to be less than 5.0%. That is

$$\frac{I_n}{I_0} > 0.95.$$

The answer to the problem can be found by solving the following equation for n:

$$\left(\cos\left(\frac{\pi}{2n}\right)\right)^{2n} = 0.95,$$

or

$$\cos\left(\frac{\pi}{2n}\right) = 0.95^{1/2n}$$

$$= (1 - 0.05)^{1/2n}.$$

We expect n ? 1, and $\pi/2n$ to be small so that we may approximate

$$\cos\left(\frac{\pi}{2n}\right) \approx 1 - \frac{\pi^2}{8n^2},$$

and

$$(1-0.05)^{1/2n} \approx 1 - \frac{0.05}{2n}.$$

We thus obtain an algebraic equation from which n can be solved, which is

$$\frac{\pi^2}{8n^2} = \frac{0.05}{2n},$$

or

$$n = \frac{\pi^2}{4 \times 0.05} = 49.3.$$

So with 50 polarizing sheets, each rotated by an angle $\pi/50$ with respect to the preceding sheet plane of vibration of polarized light can be rotated by 90^0 and at the same it can be ensured that the total intensity loss will be less than 5.0%.