689.

Problem 48.1 (RHK)

The magnetic field equations for an electromagnetic wave in free space are $B_x = B\sin(ky + \omega t)$, $B_y = B_z = 0$. *We have to find* (a) *the direction of propagation;* (b) *write the electric field equation*; *and* (c) *find whether the wave is polarized, and, if so, find its direction*.

Solution:

(a)

A plane wave characterised by angular frequency ω and propagation vector $\vec{k}$ can be described by the function

$\psi = A\sin(\omega t - \vec{k}.\vec{r})$.

The direction of propagation of the wave is that of the propagation vector $\vec{k}$.

We are given an electromagnetic wave in free space whose magnetic field equations are

$B_x = B\sin(ky + \omega t)$, and $B_y = B_z = 0$.

From these functions, we note that the propagation vector for this wave is

$\vec{k} = -\hat{j}k$, where $\hat{j}$ is the unit vector in the y-direction.

Therefore, the direction of propagation of the wave is the negative y-direction.

(b)

We know that the electric field $\vec{E}$, the magnetic field $\vec{B}$, and the Poynting vector $\vec{S}$, which is in the direction of the propagation vector $\vec{k}$, are perpendicular to each other and the definition of the Poynting vector is

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} .$$

The magnetic field of the wave is in the direction of the unit vector $\hat{i}$, and its Poynting vector is in the direction $-\hat{j}$, the electric field vector has to be in the direction $\hat{k}$.

In other words the non-zero component of the electric field of the wave will be E_z.

The Maxwell's equation connecting the electric and magnetic fields that we can use is

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}.$$

Therefore, we have

$$\frac{\partial E_z}{\partial y} = -\frac{\partial B_x}{\partial t} = -B\omega\cos\left(ky + \omega t\right),$$

and

$$E_z = -\frac{\omega}{k} B \sin\left(ky + \omega t\right) = -cB \sin\left(ky + \omega t\right),$$

as

$$\frac{\omega}{k} = c.$$

The other components of the electric field of the wave are $E_x = E_y = 0.$

(c)

As the polarization of the wave is determined by its electric field vector $\vec{E}$, the wave is linearly polarized and its polarization vector is in the $\hat{k}$ direction.