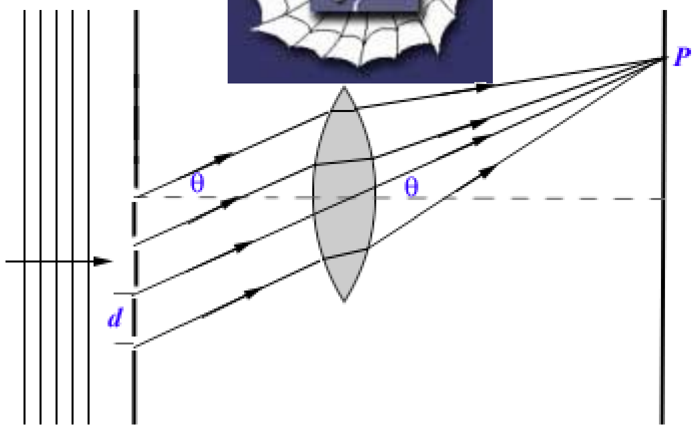
682.

<u>Problem 47.29 (RHK)</u>

A diffraction grating has a resolving power $R = \lambda/\Delta\lambda = Nm$. (a) *We have to show that the corresponding frequency range* $\Delta\nu$ *that can just be resolved in given by* $\Delta\nu = c/Nm\lambda$. *From the figure shown, we have to show that the "times of flight" of the two extreme rays differ by an amount* $\Delta t = (Nd/c)\sin\theta$; (c) *and that* $(\Delta\nu)(\Delta t) = 1$, *which is independent of the various grating parameters. We may assume* N ? 1.



<u>Solution:</u>

For light frequency ν and wavelength λ are related to each other through the speed of light *c*. We have the relation

$\nu\lambda = c.$

From this relation we note that

$$\Delta\nu\lambda + \nu\Delta\lambda = 0,$$

or

$$\left|\Delta\lambda\right| = \frac{\left|\Delta\nu\right|\lambda}{\nu}.$$

The resolving for a grating is

$$R = \frac{\lambda}{\Delta\lambda} = Nm.$$

We have

$$\frac{\left|\Delta\nu\right|\lambda}{\nu} = \frac{\lambda}{Nm},$$

or

$$\left|\Delta\nu\right| = \frac{\nu}{Nm} = \frac{c}{Nm\lambda}.$$

(b)

The path difference between rays reaching the point *P* on the screen from two successive rulings of the grating is

$$\Delta x = d\sin\theta.$$

If the grating has *N* rulings, the path difference between the extreme rays will be $\left(N-1\right)d\sin\theta$.

Assuming $N \gg 1$, we approximate it by $Nd\sin\theta$.

Therefore, the difference in "times of flight" of the extreme rays will be

$$\Delta t = \frac{Nd \sin \theta}{c}.$$

(c)

We thus find that

$$(\Delta \nu)(\Delta t) = \frac{Nd \sin \theta}{c} \times \frac{c}{Nm\lambda} = \frac{d \sin \theta}{m\lambda} = 1.$$