681.

<u>Problem 47.27 (RHK)</u>

A grating has 40,000 *rulings spread over* 76 mm. (a) *We have to estimate the dispersion in* ${}^{0}\!/\text{nm}$ *for sodium light* $(\lambda = 589\ \text{nm})$ *in the first three orders*; *and* (b) *the resolving powers in these orders*.

<u>Solution:</u>

(a)

The dispersion of a grating is defined by the relation

$$D = \frac{m}{d\cos\theta},$$

where m is the order of the spectrum and d is the spacing of the grating rulings.

As 40,000 rulings are spread over 76 mm, the grating spacing is

$$d = \frac{76\times 10^{6}\ \text{nm}}{40{,}000} = 1900\ \text{nm}.$$

The angles from the incident normal at which the lines in the first, second, and third orders for wavelength $\lambda = 589\ \text{nm}$ will be observed are as follows;

(1) $\sin\theta_1 = \dfrac{589\ \text{nm}}{1900\ \text{nm}}$, and $\theta_1 = 18.0^0$;

(2) $\sin\theta_2 = \dfrac{2\times 589\ \text{nm}}{1900\ \text{nm}}$, and $\theta_2 = 38.3^0$;

and

(3) $\sin\theta_3 = \dfrac{3\times 589\ \text{nm}}{1900\ \text{nm}}$, and $\theta_3 = 68.4^0$.

The dispersions in the first, second, and third orders for the sodium light in this grating can be calculated from the relation

$$D = \frac{\tan\theta}{\lambda}.$$

(1)

$$D_1 = \frac{\tan\theta_1}{\lambda} = \frac{\tan 18^0}{589}\ \text{rad nm}^{-1} = 5.516\times10^{-4}\ \text{rad nm}^{-1}$$

$$= \frac{5.516\times10^{-4}\times180}{\pi}\ {}^0/\text{nm}$$

$$= 0.032^0\ \text{nm}^{-1}.$$

(2)

$$D_2 = \frac{\tan\theta_2}{\lambda} = \frac{\tan 38.3^0}{589}\ \text{rad nm}^{-1} = 0.077^0\ \text{nm}^{-1}.$$

(3)

$$D_3 = \frac{\tan\theta_3}{\lambda} = \frac{\tan 68.4^0}{589}\ \text{rad nm}^{-1} = 0.25^0\ \text{nm}^{-1}.$$

(b)

The resolving power of the grating in the first, second, and third orders can be determined from the definition $R = mN$.

As $N = 40,000$, the resolving power of this grating in the first, second, and third orders will be 40,000, 80,000, and 120,000, respectively.