Problem 47.26 (RHK)

We have to show that the dispersion of a grating can be written as

$$D = \frac{\tan \theta}{\lambda}.$$

Solution:

The angular separation $\Delta\theta$ per unit wavelength $\Delta\lambda$ is called the dispersion of grating.

We will obtain the expression for the dispersion of grating by using the grating equation

$$d\sin\theta = m\lambda$$
, $m = 0, \pm 1, \pm 2,...$

We use angle θ and wavelength λ as variables, and by differentiation obtain the relation

$$d\cos\theta\Delta\theta = m\Delta\lambda$$
,

or

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} = \frac{d\sin\theta}{\lambda d\cos\theta} = \frac{\tan\theta}{\lambda} .$$