674.

Problem 47.9 (RHK)

Using the expression for the intensity pattern for a three-slit "grating":

$$I = \frac{1}{9} I_m (1 + 4\cos\phi + 4\cos^2\phi),$$

where

$$\phi = \frac{2\pi d \sin \theta}{\lambda} ,$$

We have to show (a) that a three-slit "grating" has only one secondary maximum; (b) we have to find its location and (c) its relative intensity.

Solution:

(a)

We will first locate the extremum of the function

$$I = \frac{1}{9} I_m (1 + 4\cos\phi + 4\cos^2\phi).$$

We will calculate $\frac{dI}{d\phi}$ and find the solutions of the

equation

$$\frac{dI}{d\phi} = 0,$$

or
 $(-4\sin\phi + 8\cos\phi(-\sin\phi)) = 0,$
or
 $\sin\phi(1 + 2\cos\phi) = 0.$

Therefore, the zeros of $\frac{dI}{d\phi}$ will be at

$$\phi = 0, \ \frac{2\pi}{3}, \ \pi, 2\pi,...$$

We note that the first principal maximum occurs at $\phi = 2\pi$. We will therefore examine the nature of extremum of $I(\phi)$ at $\phi = \frac{2\pi}{3}$, and $\phi = \pi$.

Condition of a local maximum of $\frac{dI}{d\phi}$ is that

$$\frac{d^2I}{d\phi^2} < 0.$$

We find that

$$\frac{d^2I}{d\phi^2} = \frac{1}{9}I_m \left(-4\cos\phi - 8\cos 2\phi\right).$$

We note that

$$\left(\frac{d^2 I}{d\phi^2}\right]_{\phi=\pi} = -\frac{4}{9}I_m < 0,$$

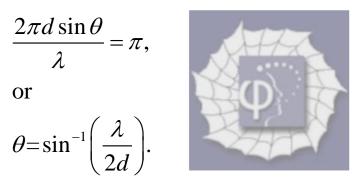
and
$$\left(\frac{d^2 I}{d\phi^2}\right]_{\phi=2\pi/3} = \frac{6}{9} > 0.$$

Therefore, the secondary maximum will occur at $\phi = \pi$.

(b)

The location of the secondary maximum will, therefore,

be at



(c)

The relative intensity of the secondary maximum will be

$$\frac{I(\pi)}{I_m} = \frac{1}{9}.$$

We note that as the principal maxima occur at the

'grating" condition

 $d\sin\theta = m\lambda, \ m = 0, \pm 1, \pm 2, \dots$

The first principal maximum will occur at

$$\sin\theta = \frac{\lambda}{d},$$

and the secondary maximum at

$$\sin\theta = \frac{\lambda}{2d}.$$

