674.

Problem 47.9 (RHK)

Using the expression for the intensity pattern for a three-slit "*grating*":

$$I = \frac{1}{9} I_m \left(1 + 4\cos\phi + 4\cos^2\phi\right),$$

where

$$\phi = \frac{2\pi d \sin\theta}{\lambda} \ ,$$

We have to show (a) *that a three-slit* "*grating*" *has only one secondary maximum*; (b) *we have to find its location and* (c) *its relative intensity*.

Solution:

(a)

We will first locate the extremum of the function

$$I = \frac{1}{9} I_m \left(1 + 4\cos\phi + 4\cos^2\phi\right).$$

We will calculate $\dfrac{dI}{d\phi}$ and find the solutions of the

equation

$$\frac{dI}{d\phi}=0,$$

or

$$\left(-4\sin\phi+8\cos\phi\left(-\sin\phi\right)\right)=0,$$

or

$$\sin\phi\left(1+2\cos\phi\right)=0.$$

Therefore, the zeros of $\frac{dI}{d\phi}$ will be at

$$\phi=0,\ \frac{2\pi}{3},\ \pi,\ 2\pi,..$$

We note that the first principal maximum occurs at $\phi=2\pi$. We will therefore examine the nature of extremum of $I\left(\phi\right)$ at $\phi=\frac{2\pi}{3}$, and $\phi=\pi.$

Condition of a local maximum of $\frac{dI}{d\phi}$ is that

$$\frac{d^2I}{d\phi^2}<0.$$

We find that

$$\frac{d^2I}{d\phi^2}=\frac{1}{9}I_m\left(-4\cos\phi-8\cos 2\phi\right).$$

We note that

$$\left(\frac{d^2 I}{d\phi^2}\right]_{\phi=\pi} = -\frac{4}{9} I_m < 0,$$

and

$$\left(\frac{d^2 I}{d\phi^2}\right]_{\phi=2\pi/3} = \frac{6}{9} > 0.$$

Therefore, the secondary maximum will occur at $\phi = \pi$.

(b)

The location of the secondary maximum will, therefore, be at

$$\frac{2\pi d \sin\theta}{\lambda} = \pi,$$

or

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right).$$

(c)

The relative intensity of the secondary maximum will be

$$\frac{I(\pi)}{I_m} = \frac{1}{9}.$$

We note that as the principal maxima occur at the 'grating" condition

$$d \sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, ...$$

The first principal maximum will occur at

$$\sin\theta = \frac{\lambda}{d},$$

and the secondary maximum at

$$\sin\theta = \frac{\lambda}{2d}.$$