673.

Problem 47.8 (RHK)

Using the expression for the intensity pattern for a three-slit “*grating*”:

$$I = \frac{1}{9} I_m \left(1 + 4\cos\phi + 4\cos^2\phi\right),$$

where

$$\phi = \frac{2\pi d \sin\theta}{\lambda} ,$$

We have to show that the half-width of the fringes for a three-slit diffraction pattern, assuming θ *small enough so that* $\sin\theta \approx \theta$*, is*

$$\Delta\theta \approx \frac{\lambda}{3.2d} .$$

Solution:

We have to find the angle θ near to the centre of the principal maximum where intensity is half of its maximum value, which is I_m. That is

$$I_\theta = \frac{1}{2} I_m .$$

We thus have the equation

$$\frac{1}{9} I_m \left(1 + 4\cos\phi + 4\cos^2\phi\right) = \frac{1}{2} I_m,$$

or

$$\left(1 + 4\cos\phi + 4\cos^2\phi\right) = \frac{9}{2},$$

or

$$\left(1+2\cos\phi\right) = \pm\frac{3}{\sqrt{2}} = \pm 2.12.$$

The physical solution is

$$\cos\phi = 0.56,$$

and

$$\phi = \cos^{-1} 0.56 = 0.976 \text{ rad.}$$

Therefore,

$$\frac{2\pi\theta d}{\lambda} \; ; \; \pm 0.976,$$

and

$$\theta = \pm\frac{0.976\lambda}{2\pi d} = \pm\frac{\lambda}{6.43d}.$$

Therefore, the half-width of the fringes for a three-slit diffraction pattern will be

$$\Delta\theta = 2\left|\theta\right| = \frac{\lambda}{3.2d}.$$