672.

<u>Problem 47.7 (RHK)</u>

We have to derive the following expression for the intensity pattern for a three-slit "grating":

$$I = \frac{1}{9} I_m \left(1 + 4\cos\phi + 4\cos^2\phi\right),$$

where

$$\phi = \frac{2\pi d \sin\theta}{\lambda} \ .$$

We may assume that $a = \lambda$.

<u>Solution:</u>

As the slit-width $a = \lambda$, where λ is the wavelength of the light incident on the three-slit "grating", we will ignore the effect on the intensity of the principal maxima due to the diffraction envelope produced because of the finite size of the slits. Further, we will assume that the magnitude of the amplitudes of the electric field vectors at the same point on the screen due to each of the three slits is equal.

We recall that the phase difference between two rays reaching a point on the screen at an angle θ from two slits separated by a distance d is related to their path difference as

$$\phi = \frac{2\pi}{\lambda} \times (\text{path difference}).$$

The path difference between rays from two successive slits is $d \sin\theta$. Therefore,

$$\phi = \frac{2\pi d \sin\theta}{\lambda} .$$

From the superposition principle the electric field at a point P at an angle θ to the incident direction will therefore be

$$\begin{aligned}
E_\theta &= E\sin(\omega t) + E\sin(\omega t + \phi) + E\sin(\omega t + 2\phi) \\
&= E\left\{\begin{matrix} \sin(\omega t) + \sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi + \\ \sin(\omega t)\cos 2\phi + \cos(\omega t)\sin 2\phi \end{matrix}\right\} \\
&= E\sin(\omega t)\left(\cos\phi + 2\cos^2\phi\right) + E\sin\phi(1 + 2\cos\phi)\cos(\omega t) \\
&= |E_\theta|\sin(\omega t + \psi),
\end{aligned}$$

where

$$|E_\theta|\cos\psi = E\left(\cos\phi + 2\cos^2\phi\right),$$

and

$$|E_\theta|\sin\psi = E\sin\phi\left(1 + 2\cos\phi\right).$$

We find that

$$|E_\theta|^2 = E^2\begin{pmatrix}\cos^2\phi + 4\cos^4\phi + 4\cos^3\phi + \sin^2\phi + \\ 4\sin^2\phi\cos^2\phi + 4\sin^2\phi\cos\phi\end{pmatrix}.$$

In the above expression using $\sin^2\phi = 1 - \cos^2\phi$, we find that

$$|E_\theta|^2 = E^2\left(1 + 4\cos\phi + 4\cos^2\phi\right).$$

The intensity of electromagnetic wave is proportional to the square of modulus of the amplitude of its electric field vector and is given by the expression

$$I_\theta = \frac{1}{2\mu_0 c}|E_\theta|^2 = \frac{1}{2\mu_0 c}E^2\left(1 + 4\cos\phi + 4\cos^2\phi\right).$$

The central maximum is at $\theta = 0$. The intensity of the central maximum will therefore be

$$I_m = \frac{1}{2\mu_0 c}|E_\theta|^2 = \frac{9E^2}{2\mu_0 c},$$

$$\therefore\ I_\theta = \frac{1}{9}I_m\left(1 + 4\cos\phi + 4\cos^2\phi\right).$$