671.

<u>Problem 47.5 (RHK)</u>

Light of wavelength 600 nm *is incident normally on a diffraction grating. Two adjacent principal maxima occur at* $\sin\theta = 0.20$ *and* $\sin\theta = 0.30$. *The fourth order is missing.* (a) *We have to find the separation between adjacent slits;* (b) *the smallest possible individual slit width; and name all orders actually occurring on the screen with the values derived in* (a) *and* (b).

<u>Solution:</u>

The grating equation is

$$d\sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, ...$$

From the statement of the problem we have two possibilities, either the adjacent principal maxima correspond to $m = 1$ and 2, or $m = 2$ and 3.

As the adjacent principal maxima occur at $\sin\theta = 0.20$ and $\sin\theta = 0.30$, they correspond to $m = 2$ and 3. For the equations

$$d \times 0.2 = 2\lambda,$$

and

$d \times 0.3 = 3\lambda$

are consistent. The wavelength of the monochromatic light is $\lambda = 600$ nm. Therefore, the slit separation is

$$d = \frac{2\times 600}{0.2}\ \text{nm} = 6000\ \text{nm} = 600\ \mu\text{m}.$$

The size of the slit width will be determined by using the information that the 4th order principal maxima is missing. The fourth order principal maximum falls at the position a diffraction minimum produced by the finite width size of slits in the grating. The intensity diffraction envelope is determined by

$$\left(\frac{\sin\alpha}{\alpha}\right)^2, \text{ where } \alpha = \frac{\pi a \sin\theta}{\lambda}.$$

The value of $\sin\theta$ for the 4th order principal maximum is

$$\sin\theta = \frac{4\times 600\ \text{nm}}{6000\ \text{nm}} = 0.4.$$

If this angle is to coincide with first diffraction minimum due to the finite width of the slits, we have the condition

$$\alpha = \pi = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi a \times 0.4}{600\ \text{nm}},$$

or

$$a = \frac{600\ \text{nm}}{0.4} = 1500\ \text{nm} = 1.5\ \mu\text{m}.$$

The second diffraction minimum will occur at

$$\sin\theta = \frac{2\lambda}{a} = \frac{2\times 600\text{ nm}}{1500\text{ nm}} = 0.8.$$

The order of the grating principal maxima that will coincide with the second diffraction minimum will be given by the condition

$$m = \frac{6000\times 0.8}{600} = 8.$$

We note that for $m = 10,\ \sin\theta = 1$. Therefore, principal maxima of order $m = 10$ and greater will not be seen on the screen.

Therefore, all orders of the principal maxima that will be seen on the screen will be

$m = 0, 1, 2, 3, 5, 7,$ and 9.