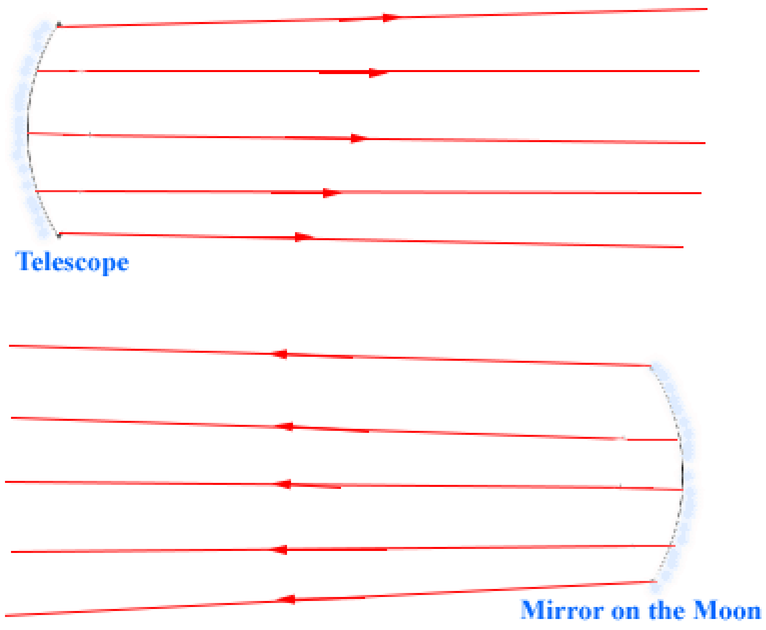
669.

Problem 37.36P (HRW)

In a joint Soviet-French experiment to monitor the Moon's surface with a light beam, pulsed radiation from a ruby laser $(\lambda = 690\ \text{nm})$ *was directed to the Moon through a reflecting telescope with a mirror radius of* 1.3 m. *A reflector on the Moon behaved like a circular plane mirror with radius* 10 cm, *reflecting the light directly back toward the telescope on Earth. The reflected light was then detected after being brought to a focus by this telescope. We have to find the fraction of the original light energy that was picked up by the detector. We may assume that for each direction of travel all the energy is in the central diffraction peak.*



Solution:

The radius of the mirror of the telescope is 1.3 m.

We know that the spread of the first diffraction peak of light waves of wavelength λ from a circular object of diameter d is given by the Rayleigh's criterion:

$$\sin\theta = \frac{1.22\times\lambda}{d} \quad (\text{first minimum: circular aperture}),$$

where θ is the angle of the first diffraction minimum from the central axis.

Therefore, the angular spread of the laser beam emerging out of the telescope mirror will be determined by the angle

$$\theta_{diff-telescope} = \frac{1.22 \times 690 \times 10^{-9}}{2.6} \text{ rad} = 3.24 \times 10^{-7} \text{ rad.}$$

The total angular spread of the laser beam will be

6.48×10^{-7} rad.

The Moon's distance from the Earth is 3.82×10^{8} m.

The angle subtended by a circular plane mirror of radius 10 cm from the Earth is

$$\theta_{mirror-moon} = \frac{20 \times 10^{-2}}{3.82 \times 10^{8}} = 5.24 \times 10^{-10}.$$

The fraction of the energy emitted by the reflecting telescope on Earth by the mirror on the Moon will therefore be

$$f_1 = \frac{5.24 \times 10^{-10}}{6.48 \times 10^{-7}} = 8.1 \times 10^{-4}.$$

The diffraction angular spread of the laser beam reflected by the mirror on the Moon will be determined by the angle

$$\theta_{diff-moon\_mirror} = \frac{1.22 \times 690 \times 10^{-9}}{20 \times 10^{-2}} = 0.42 \times 10^{-5}.$$

The angle subtended by the mirror of the telescope with respect to the mirror on the Moon will be

$$\theta_{mirror-telescope} = \frac{2.6}{3.82 \times 10^{8}} = 6.81 \times 10^{-9}.$$

The fraction of the energy reflected by the mirror on the Moon which is received by the mirror of the telescope on Earth will therefore be

$$f_2 = \frac{6.81\times10^{-9}}{2\times0.42\times10^{-5}} = 8.1\times10^{-4}.$$

The fraction of the original light energy that will be picked up by the detector on reflection from the Moon will therefore be

$$f = f_1\times f_2 = 8.1\times10^{-4}\times8.1\times10^{-4} = 6.5\times10^{-7}.$$