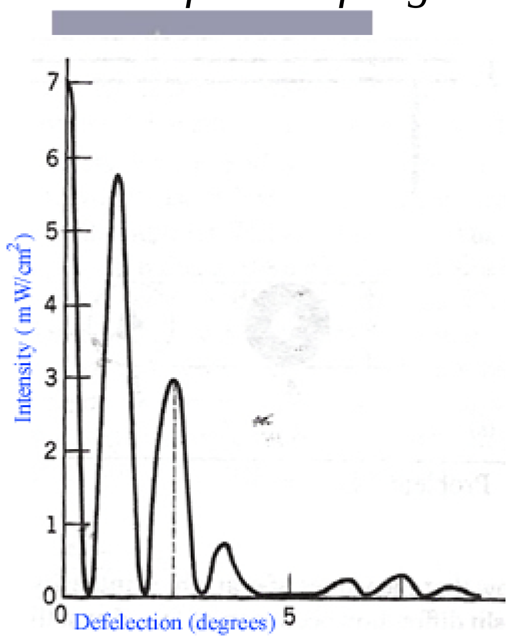
667.

<u>Problem 46.35 (RHK)</u>

Light of wavelength 440 nm *passes through a double slit, yielding the diffraction pattern of intensity I versus deflection angle* θ, *as shown in the figure. We have to calculate* (a) *the slit width and* (b) *the slit separation. We have to verify the displayed intensities of the* $m=1$ *and* $m=2$ *interference fringes.*



<u>Solution:</u>

(a)

As the boundary of the central envelope is at $\theta_c = \pm 5^0$, we note that

$$\frac{\pi a \sin|\theta_c|}{\lambda} = \pi,$$

or

$$a = \frac{\lambda}{\sin|\theta_c|}.$$

The wavelength of light used in obtaining the diffraction pattern is $\lambda = 440$ nm.

Therefore, the width *a* of the slit will be

$$a = \frac{440 \times 10^{-9}}{\sin 5^0} \text{ m} = 5.0 \times 10^{-6} \text{ m} = 5.0\ \mu\text{m}.$$

(b)

From the intensity diagram, we note that the peak of the fourth fringe falls on the diffraction minimum of the central envelope and that are only 3 interference fringes other than the central fringe on each side of the central fringe.

Let the two slits be separated by distance *d*. The 4th fringe is determined by the equation

$$\frac{d \sin\theta_c}{\lambda} = 4,$$

or

$$d = \frac{4\lambda}{\sin\theta_c} = 4 \times 5\ \mu\text{m} = 20\ \mu\text{m}.$$

We will next calculate the intensity of the fringe that corresponds to $m=1$.

We have the condition that

$$\frac{d\sin\theta}{\lambda}=1,$$

or

$$\frac{\sin\theta}{\lambda}=\frac{1}{d}=\frac{1}{20\ \mu\text{m}}.$$

For calculating the intensity we have to find the value of

$$\alpha=\frac{\pi a\sin\theta}{\lambda}=\frac{5\times\pi}{20}=\frac{\pi}{4}.$$

As the intensity at a fringe is given by

$$I_\theta=I_0\times\left(\frac{\sin\alpha}{\alpha}\right)^2,$$

we find the intensity of the fringe that corresponds to $m=1$ will be

$$I_1=I_0\left(\frac{\sin(\pi/4)}{\pi/4}\right)^2=5.67\ \text{mW cm}^{-2},$$

where we have used that

$$I_0=7.0\ \text{mW cm}^{-2}.$$

The intensity of the fringe, which corresponds to $m=2$, can be similarly calculated.

For $m=2$,

$$\frac{d \sin\theta}{\lambda} = 2,$$

or

$$\frac{\sin\theta}{\lambda} = \frac{2}{d}.$$

Value of α for $m = 2$ will be

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi \times 5 \times 2}{20} = \frac{\pi}{2}.$$

Therefore,

$$I_2 = I_0 \left(\frac{\sin(\pi/2)}{\pi/2} \right)^2 = \frac{7 \times 4}{\pi^2} \text{ mW cm}^{-2} = 2.8 \text{ mW cm}^{-2}.$$