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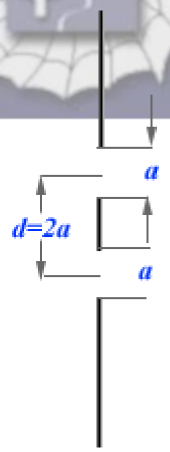
Problem 46.33 (RHK)

We consider a double-slit each of width a and let their separation $d = 2a$. (a) *We have to find the interference fringes that lie in the central diffraction envelope.* (b) *If we put* $d = a$*, the two slits coalesce into a single slit of width 2a. We have to show that the formula for the intensity of a double-slit with slits of finite size reduces to the diffraction pattern for such a slit.*



Solution:

(a)

The intensity pattern of a double-slit of slit separation *d* and size of each slit *a* is given by the equation

$$I_\theta = I_m \cos^2 \beta \times \left(\frac{\sin \alpha}{\alpha} \right)^2,$$

where

$$\beta = \frac{\pi d \sin \theta}{\lambda},$$

and

$$\alpha = \frac{\pi a \sin \theta}{\lambda}.$$

We note that the central diffraction envelope is determined by the condition that

$$\alpha = \pm \pi.$$

Therefore, the angular size of the principal diffraction envelope is

$$\theta_c = \pm \sin^{-1} \frac{\lambda}{a}.$$

Interference fringes are determined by the Young's double-slit interference condition

$$\frac{d \sin \theta}{\lambda} = m, \; m = 0, \pm 1, \pm 2, ...$$

We are given that $d = 2a$.

Therefore, for $m = \pm 1$,

we note that

$$\theta = \pm \sin^{-1} \left(\frac{\lambda}{2a} \right).$$

And, $\theta < \theta_c$, therefore two fringes corresponding to $m = \pm 1$ are contained inside the central envelope.

We note that for $m = \pm 2$, $\theta = \theta_c$, and at this point we have the diffraction minima of the central envelope.

Therefore, the total number of fringes contained inside the central envelope is three.

(b)

If $d = a$, we have

$$\beta = \alpha = \frac{\pi a \sin\theta}{\lambda}.$$

And,

$$I_\theta = I_m \left(\frac{\sin(2\alpha)}{2\alpha} \right)^2,$$

which is the intensity of diffraction of a single slit of width 2*a*.