664.

Problem 37.35 P (HRW)

A circular obstacle produces the same diffraction pattern as a circular hole of the same diameter (*except very near* $\theta = 0$). *Airborne water drops are examples of such obstacles. When we see the Moon through suspended water drops, such as in a fog, we intercept the diffraction pattern from many drops; the composite is a bright circular pattern surrounding the Moon. Next to the Moon, the pattern is white*. (a) *We have to find the colour, red or blue, that outlines the white pattern*. (b) *We may assume that the outlining ring has an angular diameter that is* 1.5 *times the angular diameter of the Moon, which is* 0.50^0. *We may also assume that the drops all have about the same diameter. We have to find the approximate value of the diameter*.

Solution:

(a)

When there are many water droplets located randomly the net effect of interference of light falling from the

Moon will be that of cumulative diffraction associated with a single droplet. Let the diameter of droplets be a m. The condition on the angular size of the diffraction ring that is the first diffraction minima is determined by the wavelength of light λ and the size of the droplet, both of which should be of the same order. The angular size of the diffraction ring will be largest for the visible light of the longest wavelength, which is red, and its wavelength is 690 nm. Therefore, the colour of the outline around the white diffraction pattern will be red.

(b)

The angular size of the diffraction ring is given to be

$$\theta = \left(\frac{0.5 \times 1.5}{2} \right)^0 = 0.375^0 = 6.54 \times 10^{-3} \text{ rad.}$$

The size of water droplets can be determined by using the result that the first diffraction minima of a circular drop of diameter a m is related to the wavelength λ by the equation

$$a \sin\theta = 1.22\lambda.$$

$\therefore$

$$a = \frac{1.22 \times 690 \times 10^{-9}}{6.54 \times 10^{-3}} \text{ m} = 128 \ \mu\text{m.}$$

The size of the water droplets will be about 128 μm.