

662.

Problem 46.25 (RHK)

A circular diaphragm 60 cm in diameter oscillates at a frequency of 25 kHz in an underwater source of sound used for submarine detection. Far from the source the sound intensity is distributed as a diffraction pattern for a circular hole whose diameter equals that of the diaphragm. Assuming that the speed of sound in water to be 1450 m s^{-1} , we have to find the angle between the normal to the diaphragm and the direction of the first minimum. (b) We have to repeat the calculation for a source having an (audible) frequency of 1.0 kHz.

Solution:

(a)

The speed of sound in water is $v = 1450 \text{ m s}^{-1}$.

The wavelength of sound waves of frequency 25 kHz will be

$$\lambda = \frac{1450}{25 \times 10^3} \text{ m} = 5.8 \times 10^{-2} \text{ m}.$$

In a diffraction from a circular hole of diameter a the position of first minima is determined by the relation

$$\theta = \frac{1.22 \times \lambda}{a}$$

It is given that the diameter of the circular diaphragm is

$$a = 60 \text{ cm} = 0.6 \text{ m.}$$

Therefore, angle θ will be

$$\theta = \frac{1.22 \times 5.8 \times 10^{-2}}{0.6} \text{ rad} = 0.1179 \text{ rad} = 6.75^\circ.$$

(b)

Let us repeat the calculation for oscillation of diaphragm with 1.0 kHz frequency. The wavelength of the sound waves will be

$$\lambda = \frac{1450}{1.0 \times 10^3} \text{ m} = 1.45 \text{ m.}$$

As $\lambda > a$, we may not be able to see the first diffraction minimum. Note

$$\frac{1.22 \times \lambda}{a} = \frac{1.22 \times 1.45}{0.6} = 2.94,$$

and cannot be satisfied for any θ .