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Problem 46.15 (RHK)

(a)*We have to show that the values of* α *at which intensity maxima for single-slit diffraction occur can be found exactly by differentiating*

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha} \right)^2$$

with respect to α *and by equating it to zero. This gives the condition*

$$\tan\alpha = \alpha.$$

(b) *The values of* α *satisfying this equation can be found by plotting graphically the curve* $y = \tan\alpha$ *and the straight line* $y = \alpha$ *and finding their intersection.* (c) *We have to find the* (*nonintegral*) *values of m corresponding to successive maxima in the single-slit pattern. We may note that the secondary maxima do not lie exactly halfway between minima.*

Solution:

(a)

As

$$\frac{I_\theta}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2,$$

the condition for maxima of the function $\left(I_\theta / I_m\right)$ is

$$\frac{d}{d\alpha}\left(I_\theta / I_m\right) = 0.$$

We have

$$2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\cos\alpha}{\alpha} - \frac{\sin\alpha}{\alpha^2}\right) = 0,$$

or

$$\left(\frac{\cos\alpha}{\alpha} - \frac{\sin\alpha}{\alpha^2}\right) = 0,$$

or

$$\tan\alpha = \alpha.$$

(b)

Solutions of the equation

$$\tan\alpha = \alpha$$

are: $\alpha = 0;$ 4.493 rad; 7.725 rad; ...

If we expect maxima to occur at midpoints of successive minima, then the locations of secondary maxima will be determined by the relation

$\alpha = \left(m + \frac{1}{2}\right)\pi,\ m = 1, 2, 3...,$

we are leaving out the central maxima, which corresponds to $\alpha = 0$.

The deviations from the midpoint approximation can be seen from first two values of α, which correspond to $m = 1$, and $m = 2$.

$m = 1$, and $\alpha = 4.493$,

$$m - \left(\frac{\alpha}{\pi} - \frac{1}{2}\right) = 1 - 0.93;$$

$m = 2$, and $\alpha = 7.725$,

$$m - \left(\frac{\alpha}{\pi} - \frac{1}{2}\right) = 2 - 1.96.$$

From the above calculations, we note that the secondary maxima lie do not lie at the midpoints of successive minima, but only closely to the midpoints.