658.

Problem 46.15 (RHK)

(a)We have to show that the values of α at which intensity maxima for single-slit diffraction occur can be found exactly by differentiating

$$I_{\theta} = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$$

with respect to α and by equating it to zero. This gives the condition

 $\tan \alpha = \alpha$.

(b) The values of α satisfying this equation can be found by plotting graphically the curve $y = \tan \alpha$ and the straight line $y = \alpha$ and finding their intersection. (c) We have to find the (nonintegral) values of m corresponding to successive maxima in the single-slit pattern. We may note that the secondary maxima do not lie exactly halfway between minima.

Solution:

(a)

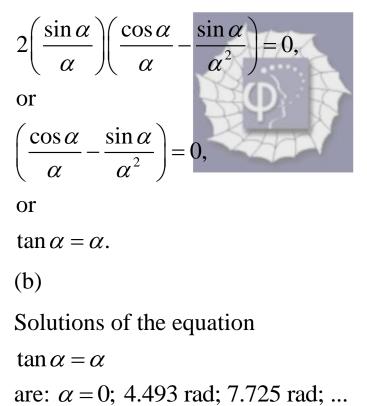
As

$$\frac{I_{\theta}}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2,$$

the condition for maxima of the function (I_{θ}/I_m) is

$$\frac{d}{d\alpha} (I_{\theta}/I_{m}) = 0.$$

We have



If we expect maxima to occur at midpoints of successive minima, then the locations of secondary maxima will be determined by the relation $\alpha = (m + \frac{1}{2})\pi, m = 1, 2, 3...,$

we are leaving out the central maxima, which

corresponds to $\alpha = 0$.

The deviations from the midpoint approximation can be seen from first two values of α , which correspond to

$$m = 1$$
, and $m = 2$.
 $m = 1$, and $\alpha = 4.493$,
 $m - \left(\frac{\alpha}{\pi} - \frac{1}{2}\right) = 1 - 0.93$;
 $m = 2$, and $\alpha = 7.725$,
 $m - \left(\frac{\alpha}{\pi} - \frac{1}{2}\right) = 2 - 1.96$.

From the above calculations, we note that the secondary maxima lie do not lie at the midpoints of successive minima, but only closely to the midpoints.