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Problem 46.13 (RHK)

We have to calculate the width of the central maximum in a single-slit diffraction pattern in which $a = 10\lambda$.

Solution:

The width of central maximum of diffraction pattern is defined to be the angular width where

$$\frac{I_\theta}{I_0} = \frac{1}{2}.$$

Also,

$$\frac{I_\theta}{I_0} = \left(\frac{\sin\alpha}{\alpha} \right)^2,$$

where

$$\alpha = \frac{\pi a \sin\theta}{\lambda}.$$

The half-angle θ which determines the width of the central maximum can therefore be determined by using the condition that

$$\sqrt{2}\sin\alpha = \alpha.$$

This equation can be solved graphically or by the method of iteration. Its solution is

$\alpha_{\frac{1}{2}} = 1.39156.$

It is given that

$\frac{a}{\lambda} = 10.$

Therefore,

$\sin\theta_{\frac{1}{2}} = \frac{\alpha\lambda}{\pi a} = \frac{1.39156}{\pi \times 10},$

and

$\theta_{\frac{1}{2}} = \sin^{-1}\left(\frac{1.39156}{\pi \times 10}\right) = 4.43 \times 10^{-2}$ rad

$= 2.53^0.$

Therefore, the angular width of the diffraction central maximum for the data given in the problem will be

$2\theta = 5.07^0.$