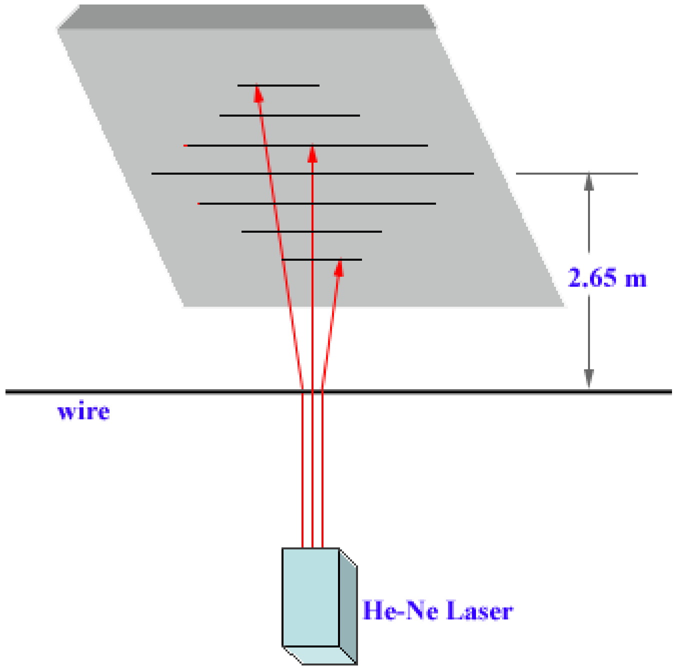
654.

Problem 46.10 (RHK)

Manufacturers of wire (*and other objects of small dimensions*) *sometimes use a laser to continually monitor the thickness of the product. The wire intercepts the laser beam, producing a diffraction pattern like that of a single slit of the same width as the wire diameter; see the figure. Suppose a He-Ne laser, wavelength* 632.8 nm, *illuminates a wire, the diffraction pattern being projected onto a screen* 2.65 m *away. If the desired wire diameter is* 1.37 mm, *we have to calculate the distance between the two tenth-order minima on each side of the central maximum.*



Solution:

From the Babinet's principle we expect that the intensity pattern on the screen due to diffraction from the wire will be equivalent to that from a slit of width equal to the diameter of the wire. The position of the minima of diffraction pattern is given by the relation

$$a\sin\theta = m\lambda,\ m = \pm1, \pm2, \pm3....,$$

where a is the diameter of the wire and λ is the wavelength of the light used for producing the diffraction pattern.

As the angles θ will be small, we use the approximation

$$\sin\theta \approx \tan\theta \approx \frac{y}{D},$$

where y is the distance of minima from the principal shadow, and D is the distance of the screen from the wire.

Therefore, the distances of the 10th minima on either side of the principal shadow will be

$$y_{10} = \frac{10\lambda D}{a},$$

and

$$y_{-10} = -\frac{10\lambda D}{a}.$$

We thus find that

$$y_{10} - y_{-10} = \frac{20\lambda D}{a} = \frac{20\times 632.8\times 10^{-9}\times 2.65}{1.37\times 10^{-3}} \text{ m}$$

$$= 24.48 \text{ mm.}$$