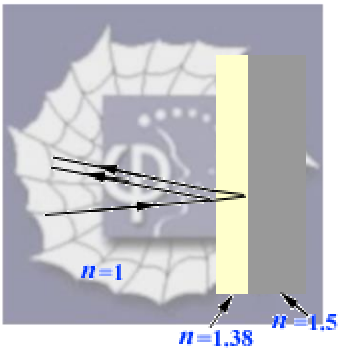
648.

Problem 45.47 (RHK)

A lens is coated with thin film of $\mathrm{MgF_2}$ $(n = 1.38)$ *to reduce the reflection from the glass surface. There is zero reflection for light of wavelength* 550 nm *at normal incidence. We have to find the factor by which the reflection is diminished at* (a) 450 nm *and* (b) 650 nm.



Solution:

We will consider interference of reflected rays one from the front surface of the coating and the other that is reflected from the backside of the coating and is refracted back into the incident medium, which is air. As the refractive index of the coating is more than that of air, there is a phase change of π on reflection. Also, the transmitted ray in the coating on reflection from the lens undergoes a phase change of π, as the refractive index of glass the material of the lens is more than that of the coating (see data shown in the figure). Therefore, both rays undergo equal amount of additional phase changes. Thee phase difference between them arises because of the additional optical path travelled by the transmitted ray inside the coating.

In the above context the condition for destructive interference is

$$2d = \left(m + 1/2\right)\lambda_{coating}, \; m = 0,1,2,3...$$

where d is the thickness of the coating. It is given that when a monochromatic light of wavelength 550 nm is incident normally there is zero reflection. We use this information for finding the thickness of the coating. We note that a solution for zero reflection is that with $m = 0$ the two reflected waves have a phase difference of π.

We thus find that

$$2d = \frac{\lambda_{coating}}{2} = \frac{\lambda}{2n_{coating}} = \frac{550}{2 \times 1.38} \text{ nm},$$

or

$$d = \frac{550}{4 \times 1.38} \text{ nm} = 99.6 \text{ nm}.$$

If ϕ is the phase difference between the waves that interfere, the intensity of the resultant is proportional to $\cos^2 \frac{\phi}{2}$.

We next calculate the phase difference for two cases of monochromatic light of wavelengths 450 nm and 650 nm.

$$\phi = 2\pi \times \frac{2d}{\lambda_{coating}} = \frac{4\pi n_{coating} d}{\lambda} = \frac{\pi \times 550}{\lambda(\text{nm})}.$$

(a)

$\lambda = 450$ nm

The phase difference will be

$$\phi = \frac{\pi \times 550}{450} \text{ rad} = 3.839 \text{ rad} = 219.9^0,$$

and the factor by which the reflection is diminished is

$$\cos^2 \frac{219.9}{2} = 0.11.$$

(b)

$\lambda = 650$ nm

The phase difference will be

$$\phi = \frac{\pi \times 550}{650} \text{ rad} = 2.658 \text{ rad} = 152.3^0,$$

and the factor by which the reflection is diminished is

$$\cos^2 \frac{152.3}{2} = 0.057.$$