635.

<u>Problem 45.25 (RHK)</u>

We have to show that the half-width $\Delta\theta$ of the double-slit interference fringes is given by

$$\Delta\theta = \frac{\lambda}{2d} \ ,$$

if θ is small enough so that $\sin\theta \, ; \, \theta$. The half-width is the angle between the two points in the fringe where the intensity is one-half that at the centre of the fringe.

<u>Solution:</u>

We first work out the expression for the intensity of the resultant of two coherent waves with phase difference ϕ.

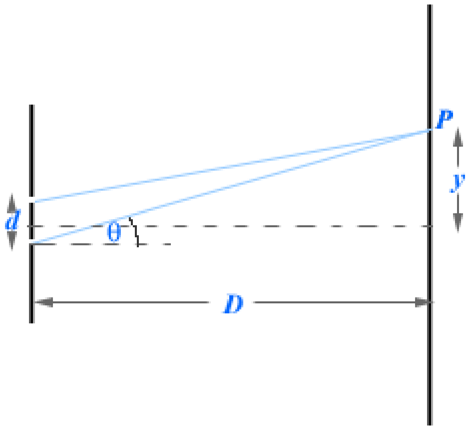
Let the electric field components of the two waves at a point *P* at time t be described by the functions

$E_1 = E_0 \sin(\omega t)$,

and

$E_2 = E_0 \sin(\omega t + \phi)$,

where $\omega (= 2\pi\nu)$ is the angular frequency of the waves and ϕ is the phase difference between them. We note that ϕ depends upon the location of the point P, which is described by the angle θ in a double-slit experiment.



We have

$$E = E_1 + E_2$$
$$= E_\theta \sin\left(\omega t + \beta\right),$$

where the phase β is

$$\beta = \tfrac{1}{2}\phi,$$

and the amplitude is

$$E_\theta = 2E_0 \cos\beta.$$

As the intensity I is proportional to the square of the amplitude, we note that

$$I\left(\theta\right) = 4I_0 \cos^2\left(\frac{\phi}{2}\right),$$

and for small θ

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \; ; \; \frac{2\pi}{\lambda} d\theta.$$

And

$$I\left(\theta\right) = 4I_0 \cos^2\left(\frac{\pi d\theta}{\lambda}\right).$$

Let the centre of the m th- fringe be at angle θ_m. We have

$\frac{d\theta_m}{\lambda} = m,$ where m is an integer. We note that

$$I(\theta_m) = 4I_0$$

Let the intensity is one-half that at the centre of the fringe at $\theta_m + \Delta\theta$. We therefore have

$$I(\theta_m + \Delta\theta) = 2I_0.$$

This implies that

$$\cos^2\left(\frac{\pi d}{\lambda}(\theta_m + \Delta\theta)\right) = \frac{1}{2},$$

or

$$\frac{1}{2}\left\{\cos\left(\frac{2\pi d}{\lambda}\theta_m + \frac{2\pi d}{\lambda}\Delta\theta\right) + 1\right\} = \frac{1}{2},$$

or

$$\cos\left(\frac{2\pi d}{\lambda}\Delta\theta\right) = 0.$$

This implies that

$$\frac{2\pi d}{\lambda}\Delta\theta = \pm\frac{\pi}{2},$$

or

$$\Delta\theta = \pm\frac{\lambda}{4d}.$$

The half-width is the angle between the two points in the fringe where the intensity is one-half that at the centre of the fringe. Therefore, half-width of the fringe will be

$$2\left|\Delta\theta\right| = \frac{\lambda}{2d}.$$