621.

Problem 44.29 (RHK)

(a) *We have to show that a thin converging lens of focal length f followed by a thin diverging lens of focal length –f will bring parallel light to a focus beyond the second lens provided the separation of the lenses L satisfies* $0 < L < f$. (b) *We have to find whether this property changes if the lenses are interchanged.* (c) *We have to answer what happens when* $L = 0$.

Solution:

The separation between the two lenses is *L*. As the parallel light strikes first the converging lens of focal length *f*, a real image will be formed at the right of the converging lens at a distance *f* from the lens. If $0 < L < f$, the image formed by the first lens will be to the right of the diverging lens, whose focal length is –*f*. The object distance with respect to the second lens will be $-(f - L)$ as it sees that the converging rays from the first lens which strike it are coming from a virtual source to the right of it. The image distance i' can therefore be determined from the thin lens formula. We have

$$-\frac{1}{(f-L)}+\frac{1}{i'}=-\frac{1}{f},$$

or

$$i'=\frac{f(f-L)}{L}=f\left(\frac{f}{L}-1\right).$$

We have assumed that $f/L>1$. Therefore, as i' is positive, the final image formed by the combination of two lenses will be real and its location will be to the right of the second lens.

(b)

We consider next the image formation when the converging and the diverging lenses have been interchanged. The diverging lens of focal length $-f$ will focus incident parallel light as a virtual image at a distance f to the left of it. The second lens which is a converging lens of focal length f is placed at a distance L to the right of the diverging lens. With respect to the converging lens in the sign convention that we are using the object distance will be $(L+f)$, as the object is on the *V*-side and diverging beam strike it. Therefore, the image will be formed at distance i'', which can be determined using the thin lens formula. We have

$$\frac{1}{(L+f)} + \frac{1}{i''} = \frac{1}{f},$$

or

$$i'' = \frac{f(L+f)}{L},$$

which is positive. Therefore, the final image will be real and its location will be at the right of the second lens.

(c)

When $L = 0$, the parallel light will be transmitted unchanged, as effectively the two lens combination will function as a thin parallel glass slab.