Problem 44.25 (RHK)

A luminous object and a screen are a fixed distance D apart. (a) *We have to show that a converging lens of focal length f will form a real image on the screen for two positions of the object that are separated by*

$$d = \sqrt{D\left(D - 4f\right)}\ .$$

(b) *We have to show that the ratio of the two image sizes for these two positions is*

$$\left(\frac{D-d}{D+d}\right)^2 .$$

Solution:

For answering this problem we will use the thin lens formula

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f},$$

and the expression for the lateral magnification

$$m = -\frac{i}{o},$$

where o is the object distance, i the image distance and f the focal length of the lens.

(a)

As we are considering real image formed by the lens, the image distance, i , will be positive. The object distance, o, will be $D-i$, as the object is at a distance D from the screen where the image is formed. Let the focal length of the converging lens be f. We thus have the equation

$$\frac{1}{D-i}+\frac{1}{i}=\frac{1}{f},$$

or

$$i^2-Di+fD=0.$$

The roots of this quadratic equation are

$$i=\frac{D\pm\sqrt{D^2-4fD}}{2}.$$

The object distances for the two image positions will therefore be

$$o_1=D-\frac{D+\sqrt{D^2-4fD}}{2},$$

$$o_2=D-\frac{D-\sqrt{D^2-4fD}}{2}.$$

We thus find that the distance between the two object positions for which real images are formed on the screen will be

$$d = o_1 - o_2 = \sqrt{D^2 - 4fD} = \sqrt{D\left(D - 4f\right)}.$$

(b)

The ratio of the two image sizes for these two positions of the object will therefore be

$$= \frac{i_2}{o_2} \times \frac{o_1}{i_1} = \frac{D - \sqrt{D^2 - 4fD}}{D + \sqrt{D^2 - 4fD}} \times \frac{D - \sqrt{D^2 - 4fD}}{D + \sqrt{D^2 - 4fD}}$$

$$= \frac{\left(D - d\right)^2}{\left(D + d\right)^2}.$$