

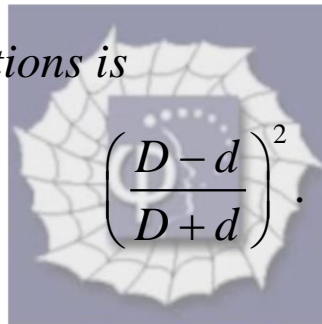
618.

**Problem 44.25 (RHK)**

*A luminous object and a screen are a fixed distance  $D$  apart. (a) We have to show that a converging lens of focal length  $f$  will form a real image on the screen for two positions of the object that are separated by*

$$d = \sqrt{D(D - 4f)} .$$

*(b) We have to show that the ratio of the two image sizes for these two positions is*


$$\left( \frac{D - d}{D + d} \right)^2 .$$

**Solution:**

For answering this problem we will use the thin lens formula

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f},$$

and the expression for the lateral magnification

$$m = -\frac{i}{o},$$

where  $o$  is the object distance,  $i$  the image distance and  $f$  the focal length of the lens.

(a)

As we are considering real image formed by the lens, the image distance,  $i$ , will be positive. The object distance,  $o$ , will be  $D - i$ , as the object is at a distance  $D$  from the screen where the image is formed. Let the focal length of the converging lens be  $f$ . We thus have the equation

$$\frac{1}{D-i} + \frac{1}{i} = \frac{1}{f},$$

or

$$i^2 - Di + fD = 0.$$

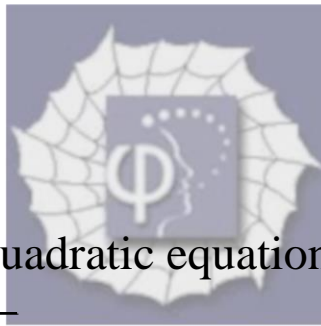
The roots of this quadratic equation are

$$i = \frac{D \pm \sqrt{D^2 - 4fD}}{2}.$$

The object distances for the two image positions will therefore be

$$o_1 = D - \frac{D + \sqrt{D^2 - 4fD}}{2},$$

$$o_2 = D - \frac{D - \sqrt{D^2 - 4fD}}{2}.$$



We thus find that the distance between the two object positions for which real images are formed on the screen will be

$$d = o_1 - o_2 = \sqrt{D^2 - 4fD} = \sqrt{D(D - 4f)}.$$

(b)

The ratio of the two image sizes for these two positions of the object will therefore be

$$\begin{aligned} \frac{i_2}{o_2} \times \frac{o_1}{i_1} &= \frac{D - \sqrt{D^2 - 4fD}}{D + \sqrt{D^2 - 4fD}} \times \frac{D - \sqrt{D^2 - 4fD}}{D + \sqrt{D^2 - 4fD}} \\ &= \frac{(D - d)^2}{(D + d)^2}. \end{aligned}$$

