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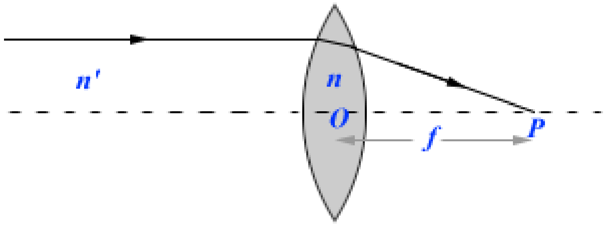
<u>Problem 44.15 (RHK)</u>

We have to show that the focal length f for a thin lens whose index of refraction is n and which is immersed in a fluid whose index of refraction is n' *is given by*

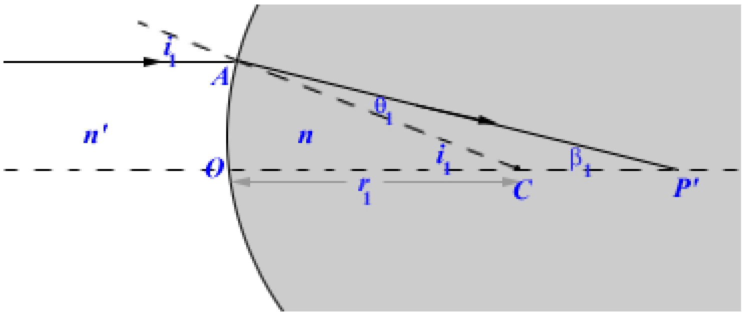
$$\frac{1}{f} = \frac{n - n'}{n'}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

<u>Solution:</u>

We will derive the result by considering refraction at two curved surfaces of radius of curvature $|r_1|$ and $|r_2|$, respectively. For determining the focal length of the lens we consider a ray parallel to the optic axis and find the distance from the lens where this ray after undergoing two refractions crosses the optic axis. We will calculate the focal length *f* in thin lens approximation. Without loss of generality, we assume that the index of refraction of the glass *n* is less than the index of refraction n' of the medium in which the lens is immersed.



For showing refractions at the two curved surfaces we will draw enlarged diagrams, as shown below.



Let the angle of incidence of the ray at A be i_1. By Snell's law the angle of refraction θ_1 and the angle of incidence i_1 are related as

$$n' \sin i_1 = n \sin \theta_1 \ .$$

For a paraxial ray and in thin lens approximation we will assume that both angles i and θ_1 are small, and we can use the approximations

$$\sin i_1 \; ; \; i_1,$$

and

$$\sin \theta_1 \; ; \; \theta_1 \ .$$

In the triangle $\Delta C_1AP'$ the exterior angle i will be equal to the sum of angles θ_1 and β_1. We have

$$i_1 = \theta_1 + \beta_1 \ .$$

From Snell's law relation, we note that

$$\theta_1 \ ; \ \frac{n'}{n} i_1 \ .$$

We therefore have the relation

$$\left(\frac{n-n'}{n}\right) i_1 = \beta_1 .$$

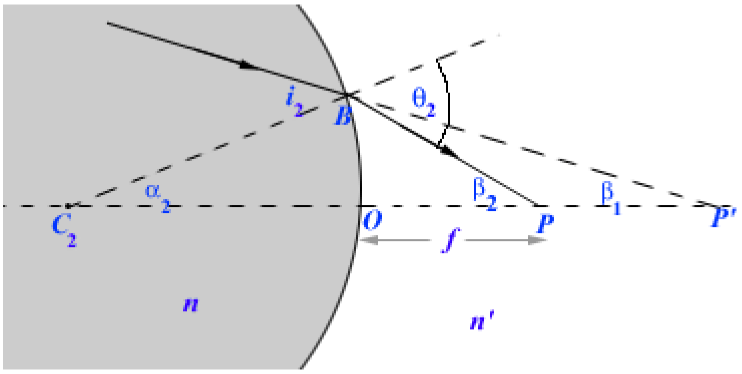
As

$$i_1 = \frac{OA}{OC_1}, \text{ and } \beta_1 \ ; \ \frac{OA}{OP'},$$

we get the relation

$$\frac{1}{OP'} = \left(\frac{n-n'}{n}\right) \frac{1}{OC_1} \ .$$

We consider next the refraction at the second curved surface, whose centre of curvature is C_2 and the radius of curvature is OC_2.



In this case Snell's law connects angle of incidence i_2 and the angle of refraction θ_2. We have

$$n \sin i_2 = n' \sin \theta_2 \ .$$

In the small angle approximation, we get

$$n i_2 \ ; \ n'\theta_2 .$$

As i_2 is the exterior angle of $\Delta BC_2P'$, we have

$$i_2 = \alpha_2 + \beta_1 \ .$$

And, as θ_2 is the exterior angle of triangle ΔPBC_2, we have

$$\theta_2 = \alpha_2 + \beta_2 \ .$$

Using the result $n i_2 \ ; \ n'\theta_2$, we have

$$n\alpha_2 + n\beta_1 = n'\alpha_2 + n'\beta_2,$$

or

$$\beta_2 = \left(\frac{n}{n'} - 1 \right) \alpha_2 + \frac{n}{n'} \beta_1.$$

In the small angle approximation, we thus get

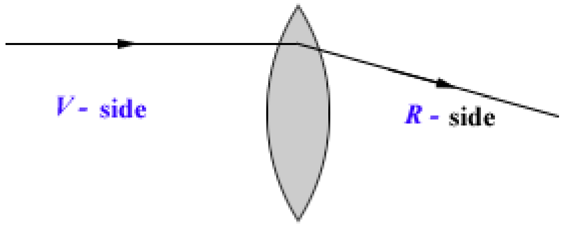
$$\frac{OB}{OP}=\left(\frac{n}{n'}-1\right)\frac{OB}{OC_2}+\frac{n}{n'}\frac{OB}{OP'},$$

or

$$\frac{1}{OP}=\left(\frac{n}{n'}-1\right)\frac{1}{OC_2}+\left(\frac{n}{n'}-1\right)\frac{1}{OC_1}$$

$$=\left(\frac{n-n'}{n'}\right)\left(\frac{1}{OC_1}+\frac{1}{OC_2}\right).$$

We use the sign convention as shown in the following figure:



As C_1 is in the R-side, r_1 is positive and $OC_1=r_1$; and as C_2 is in the V-side, r_2 is negative and $OC_2=-r_2$. Also, by definition, $OP=f$, the focal length of the thin lens. We therefore have the result

$$\frac{1}{f}=\frac{n-n'}{n'}\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$