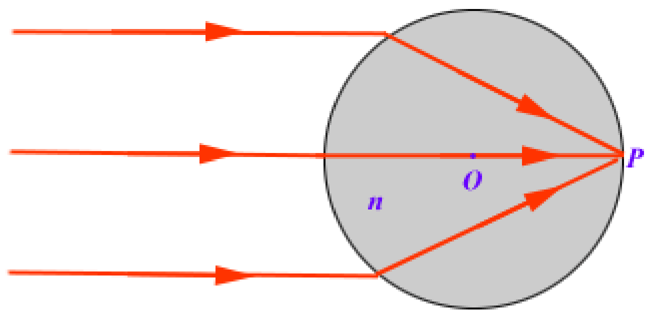
610.

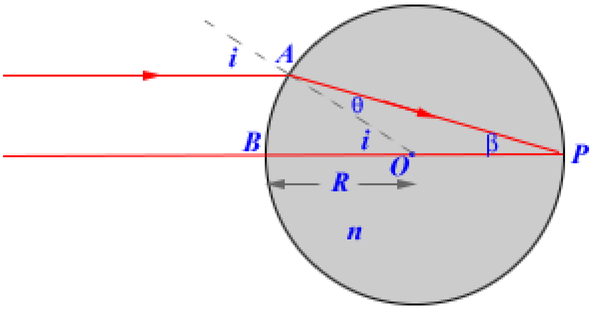
Problem 44.9 (RHK)

A parallel beam of light from a laser falls on a solid transparent sphere of index of refraction n, as shown in the figure. (a) *We have to show that the beam cannot be brought to a focus at the back of the sphere unless the beam width is small compared with the radius of the sphere.* (b) *If the condition in* (a) *is satisfied, we have to find the index of refraction of the sphere.* (c) *We have to find the index of refraction, if any, that will focus the beam at the centre of the sphere.*



Solution:

(a)



We consider a beam with half-width *AB*, as shown in the figure. The ray that strikes the spherical surface at A is considered so that it is focused at P , the point at the back of the sphere. From the line diagram, we note that the incident angle i, angle of refraction θ and the angle β that the refracted ray makes with the axis at the point P are related as

$$i = \theta + \beta.$$

As the index of refraction of the material of the spherical block is *n*, we have from Snell's law

$$\sin i = n \sin \theta.$$

Also, we note that

$$i = \frac{AB}{R},\ \text{and}\ \beta \,;\ \frac{AB}{2R}.$$

If $AB =\ R$, the approximation will be better and the incident angles and angle of refraction will be small.

Beam with small half-width compared to the radius of curvature R can be focussed at P , as then

$i\ ;\ n\theta$,

and the following condition for focussing of the beam can be satisfied:

$$i = \theta + \beta$$

$$= \frac{i}{n} + \beta.$$

(b)

This condition fixes the value of the index of refraction n for which the beam with half-width AB will get focussed at P. It will be given by the relation

$$\frac{(n-1)AB}{nR} = \frac{AB}{2R},$$

or

$$n = 2\ .$$

(c)

The condition for the beam to be focussed at the centre of curvature of the sphere is that the angle of refraction should be equal to zero. That is

$$\theta = 0.$$

As

$\theta = \dfrac{i}{n}$, this condition cannot be satisfied for any finite value of n.