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<u>Problem 44.5 (RHK)</u>

A short linear object of length L lies on the axis of a spherical mirror, a distance o from the mirror. (a) *We have to show that its image will have a length* L', *where*

$$L' = L\left(\frac{f}{o-f}\right)^2 .$$

(b) *We have to show that the longitudinal magnification* $m'\left(= L'/L\right)$ *is equal to* m^2, *where m is the lateral magnification*.

<u>Solution:</u>

For answering this problem, we will use the mirror equation

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}.$$

In this equation o is the object distance, i is the image distance, and f is the focal length which is one-half of the radius of curvature of the mirror. We use the sign convention of the textbook Physics, Halliday, Resnick and Krane.

We will find the longitudinal magnification by calculating the difference between the image distances of points at object distances $(o+L/2)$ and $(o-L/2)$. From the mirror equation, we note that

$$i(o)=\frac{of}{o-f} \; .$$

Therefore,

$$i(o+L/2)=\frac{(o+L/2)\,f}{(o+L/2)-f},$$

and

$$i(o-L/2)=\frac{(o-L/2)\,f}{(o-L/2)-f}.$$

We obtain

$$\begin{aligned} L' &= \left|i(o+L/2)-i(o-L/2)\right| \\ &= \left|\frac{(o+L/2)\,f}{(o+L/2)-f}-\frac{(o-L/2)\,f}{(o-L/2)-f}\right| \\ &= \frac{Lf^2}{(o+L/2-f)(o-L/2+f)} \\ &= \frac{Lf^2}{\left((o-f)^2-L^2/4\right)}. \end{aligned}$$

As the object is assumed to be short, we use the approximation

$$\frac{L^2}{4(o-f)^2} = 1,$$

and find

$$L' \approx \frac{Lf^2}{(o-f)^2}.$$

Therefore, the longitudinal magnification is

$$m' = \left(\frac{L'}{L}\right) = \left(\frac{f}{o-f}\right)^2.$$

The lateral magnification m is

$$m = -\frac{i}{o} = -\left(\frac{f}{o-f}\right),$$

we note that

$$m' = m^2.$$