607.

Problem 44.4 (RHK)

A luminous point is moving at speed v_0 *toward a spherical mirror, along its axis.* (a) *We have to show that the speed at which the image of this point object is moving is given by*

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0.$$

Assuming that the mirror is concave, with $r = 15\ \text{cm}$ *and that* $v_o = 5.0\ \text{cm s}^{-1}$, *we have to find the speed of the image* (b) *if the object is far outside the focal point* $(o = 75\ \text{cm})$; (c) *if it is close to the focal point* $(o = 7.7\ \text{cm})$; *and* (d) *if it is very close to the mirror* $(o = 0.15\ \text{cm})$.

Solution:

Relation between the object distance, o, image distance, i, and the radius of curvature, *r*, for a spherical mirror is given by the equation

$$\frac{1}{o}+\frac{1}{i}=\frac{2}{r},$$

where we use the sign convention of Halliday Resnick and Krane.

We note from the spherical mirror equation that

$$i=\frac{or}{2o-r}.$$

Therefore, the speed of the image, v_i, is the time derivative of the image distance i.

We thus find

$$v_i=\frac{di}{dt}=\frac{rv_o}{2o-r}-\frac{vr}{\left(2o-r\right)^2}\times 2v_o$$

$$=-\left(\frac{r}{2o-r}\right)^2 v_o,$$

$$v_o=\frac{do}{dt}.$$

(a)

We are given that $r=15$ cm, and $v_o=5.0$ cm s^{-1}.

If the object is far outside the focal point $\left(o=75\text{ cm}\right)$,

$$v_i=-\left(\frac{r}{2o-r}\right)^2 v_0=-\left(\frac{15}{150-15}\right)^2\times 5.0\text{ cm s}^{-1}$$

$$=-0.062\text{ mm s}^{-1}.$$

(b)

If the object is close to the focal point $\left(o = 7.7 \text{ cm}\right)$

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0 = -\left(\frac{15}{15.4 - 15}\right)^2 \times 5.0 \text{ cm s}^{-1}$$

$$= -70.3 \text{ m s}^{-1}.$$

(c)

If the object is very close to the mirror $\left(o = 0.15 \text{ cm}\right)$,

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0 = -\left(\frac{15}{0.30 - 15}\right)^2 \times 5.0 \text{ cm s}^{-1}$$

$$= -5.21 \text{ cm s}^{-1}.$$