

607.

Problem 44.4 (RHK)

A luminous point is moving at speed v_0 toward a spherical mirror, along its axis. (a) We have to show that the speed at which the image of this point object is moving is given by

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0.$$

Assuming that the mirror is concave, with $r = 15$ cm and that $v_0 = 5.0$ cm s⁻¹, we have to find the speed of the image (b) if the object is far outside the focal point ($o = 75$ cm); (c) if it is close to the focal point ($o = 7.7$ cm); and (d) if it is very close to the mirror ($o = 0.15$ cm).

Solution:

Relation between the object distance, o , image distance, i , and the radius of curvature, r , for a spherical mirror is given by the equation

$$\frac{1}{o} + \frac{1}{i} = \frac{2}{r},$$

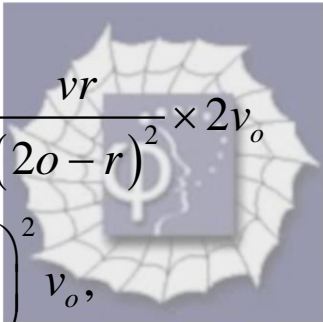
where we use the sign convention of Halliday Resnick and Krane.

We note from the spherical mirror equation that

$$i = \frac{or}{2o - r}.$$

Therefore, the speed of the image, v_i , is the time derivative of the image distance i .

We thus find

$$\begin{aligned} v_i &= \frac{di}{dt} = \frac{rv_o}{2o - r} - \frac{vr}{(2o - r)^2} \times 2v_o \\ &= -\left(\frac{r}{2o - r}\right)^2 v_o, \end{aligned}$$


$$v_o = \frac{do}{dt}.$$

(a)

We are given that $r = 15$ cm, and $v_o = 5.0$ cm s⁻¹.

If the object is far outside the focal point ($o = 75$ cm),

$$\begin{aligned} v_i &= -\left(\frac{r}{2o - r}\right)^2 v_o = -\left(\frac{15}{150 - 15}\right)^2 \times 5.0 \text{ cm s}^{-1} \\ &= -0.062 \text{ mm s}^{-1}. \end{aligned}$$

(b)

If the object is close to the focal point ($o = 7.7 \text{ cm}$)

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0 = -\left(\frac{15}{15.4 - 15}\right)^2 \times 5.0 \text{ cm s}^{-1}$$
$$= -70.3 \text{ m s}^{-1}.$$

(c)

If the object is very close to the mirror ($o = 0.15 \text{ cm}$),

$$v_i = -\left(\frac{r}{2o - r}\right)^2 v_0 = -\left(\frac{15}{0.30 - 15}\right)^2 \times 5.0 \text{ cm s}^{-1}$$
$$= -5.21 \text{ cm s}^{-1}.$$

