603.

Problem 43.49 (RHK)

In an optical fibre, different rays travel different paths along the fibre, leading to different travel times. This causes a light pulse to spread out as it travels along the fibre, resulting in information loss. The delay time should be minimised in designing a fibre. Consider a ray that travels a distance L along a fibre axis and another that is reflected, at the critical angle, as it travels to the same distance as the first. (a) *We have to show that the difference Δt in the time of arrivals is given by*

$$\Delta t = \frac{L}{c}\frac{n_1}{n_2}\left(n_1 - n_2\right),$$

where n_1 is the index of refraction of the core and n_2 is the index of refraction of the cladding. (b) *We have to evaluate Δt for the fibre in which $n_1 = 1.58$ and $n_2 = 1.53$, with $L = 350\ \text{km}$.*

Solution:

We will refer to the problem **602**, particularly to the figure in it for answering this problem.

As the refractive index of core of the fibre cable is n_1 the time taken by a light pulse in travelling a distance L along its axis will be

$$t_1 = \frac{L}{v} = \frac{L}{c/n_1} = \frac{n_1 L}{c}.$$

As the refractive index of the cladding is n_2, the critical angle θ_c is given by

$$\sin\theta_c = \frac{n_2}{n_1}.$$

A pulse that moves inside the fibre by total internal reflection covers a distance L along the axis by travelling distance

$$L' = \frac{L}{\sin\theta_c} = \frac{n_1 L}{n_2}.$$

Therefore, the travel time for the pulse that moves inside the cable through total internal reflections will be

$$t_2 = \frac{L'}{v} = \frac{n_1 L}{n_2} \times \frac{n_1}{c} = \frac{n_1{}^2 L}{n_2 c}.$$

Hence, the time difference Δt in the arrival of the pulse that travels through total internal reflection and of the pulse that travels along the axis of the cable will be

$$\Delta t = t_2 - t_1 = \frac{n_1^2 L}{n_2 c} - \frac{n_1 L}{c} = \frac{L}{c} \times \frac{n_1}{n_2}\left(n_1 - n_2\right).$$

(b)

We calculate Δt for the data of the problem:

$L = 350$ km; $n_1 = 1.58$ and $n_2 = 1.53$.

$$\Delta t = \frac{350 \times 10^3}{3 \times 10^8} \times \frac{1.58}{1.53}\left(1.58 - 1.53\right) \text{ s}$$
$$= 60.2\ \mu\text{s}.$$