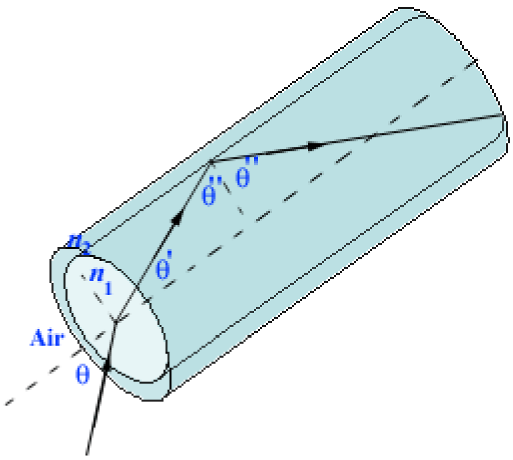
Problem 43.48 (RHK)

A particular optical fibre consists of a non-graded glass core (index of refraction n_1) *surrounded by a cladding (index of refraction* $n_2 < n_1$). *Suppose that a beam of light enters the fibre from air at angle* θ *with the fibre axis as shown in the figure*. (a) *We have to show that the greatest possible value of* θ *for which a ray can be propagated down the fibre is given by*

$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}\ .$$

(b) *Assuming that the indices of refraction of the glass and the coating are* 1.58 *and* 1.53, *respectively, we have to calculate the value of this angle*.



Solution:

(a)

As shown in the figure, let a beam of light enter the optical fibre from air at incident angle θ. The angle of refraction θ' in the glass is related to θ by the Snell's law. Let the index of refraction of the glass be n_1. We have

$$\sin\theta = n_1 \sin\theta'.$$

This beam hits the glass-coating interface at angle of incidence, $\theta'' = \frac{\pi}{2} - \theta'$. For θ'' greater than the critical angle θ_c, determined by the condition that

$$\sin\theta_c = \frac{n_2}{n_1},$$

beam that enters the optical fibre from air will travel by undergoing total internal reflection. Therefore, the maximum possible value of the angle θ with which the beam can enter the optical fibre from air so that it travels inside the optical fibre through process of total internal reflection will be given by

$$\sin\theta_{\max} = n_1 \sin\left(\frac{\pi}{2} - \theta_c\right) = n_1 \cos\theta_c,$$

Or

$$\sin\theta_{\max} = n_1\sqrt{1 - \sin^2\theta_c} = n_1\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$= \sqrt{n_1^2 - n_2^2} \;.$$

And, therefore,

$$\theta_{\max} = \sin^{-1}\sqrt{n_1^2 - n_2^2} \;.$$

(b)

For $n_1 = 1.58$, and $n_2 = 1.53$, we find

$$\theta_{\max} = \sin^{-1}\sqrt{1.58^2 - 1.53^2} = 22.59^0.$$