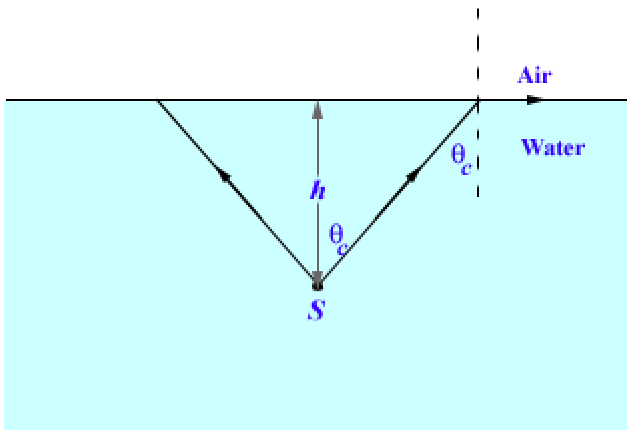
601.

<u>Problem 43.47 (RHK)</u>

A point source of light is placed a distance h below the surface of a large deep lake. (a) *We have to show that the fraction of the light energy that escapes directly from the water surface is independent of h and is given by*

$$f = \frac{1}{2}\left(1 - \sqrt{1 - 1/n^2}\right),$$

where n is the index of refraction of water. We will assume that there is no absorption within the water and reflection at the surface, except where it is total. (b) *We will evaluate this fraction numerically.*



Solution:

(a)

Let n be the index of refraction of water. From Snell's law we calculate the critical angle, θ_c, for which the refracted ray is along the air-water surface. Rays from the source S, with angle of incidence greater than θ_c, undergo total internal reflection and remain within the water.

We have

$$n \sin \theta_c = 1,$$

or

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right).$$

Therefore, the light energy from the source S that escapes from water will be contained within the cone of half-angle θ_c. We calculate the fraction of the energy that escapes from water by calculating the area on a sphere of radius h subtended by cone of half-angle θ_c and dividing it by the area of a sphere of radius h, that is $4\pi h^2$.

We have,

$$f = \frac{1}{4\pi h^2}\int_0^{\theta_c} 2\pi h \sin\theta\left(h d\theta\right)$$

$$= \frac{1}{2}\left(1-\cos\theta_c\right) = \frac{1}{2}\left(1-\sqrt{1-\sin^2\theta_c}\right)$$

$$= \frac{1}{2}\left(1-\sqrt{1-\frac{1}{n^2}}\right).$$

We note that *f* is independent of *h*, the depth at which the source is placed in the medium, whose refractive index is greater than that of the medium of the refracted light.

(b)

Taking for the refractive index of water the value

$n = 1.33$,

the numerical value of the fraction *f* will be

$$f = \frac{1}{2}\left(1-\sqrt{1-1/1.33^2}\right) = 0.170 \ .$$