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Problem 43.23 (RHK)

Muons ($mass = 106\ \text{MeV}/c^2$) *and neutral pions* ($mass = 135\ \text{MeV}/c^2$), *each with momentum* $145\ \text{MeV}/c$, *pass through a transparent material. We have to find the range of index of refraction so that only the muons emit Cerenkov radiation.*

Solution:

Relativistic equation for the momentum of a particle of rest mass m moving with velocity v is

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

Algebraically rearranging the above expression, we find that

$$\frac{c}{v} = \frac{\left(m^2c^4 + p^2c^2\right)^{1/2}}{pc}.$$

We will use the above result for calculating the ratios c/v for neutral pions and muons having momentum $p = 145\ \text{MeV}/c$.

We find that

$$\left(\frac{c}{v}\right)_{muon} = \frac{\left(106^2 + 145^2\right)^{1/2}}{145} = 1.24,$$

and

$$\left(\frac{c}{v}\right)_{pion} = \frac{\left(135^2 + 145^2\right)^{1/2}}{145} = 1.37.$$

A particle emits Cerenkov radiation in a medium only if its speed exceeds the speed of light in that medium. Therefore, the range of index of refraction of the material

$$n = \frac{c}{v_{light}},$$

will be fixed by requiring that the speed of the muons is greater than the speed of light in the medium, v_{light}, and that the speed of light in the medium exceeds the speed of neutral pions in the medium. The range of refractive index of the material should therefore be $1.24 < n < 1.37$.