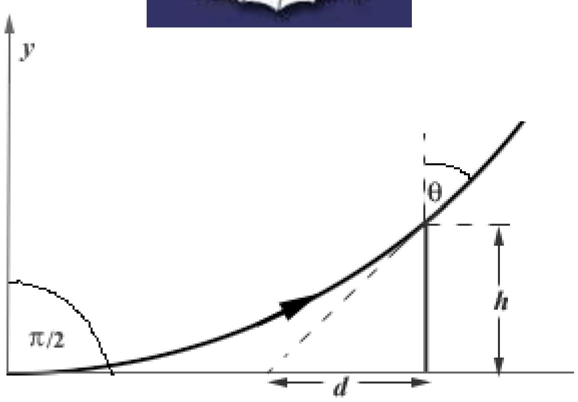
595.

Problem 43.21 (RHK)

Assume that a person is standing at one end of a long airport runway. A vertical temperature gradient in the air has resulted in the index of refraction of the air above the runway to vary with height y according to $n = n_0(1 + ay)$, *where* n_0 *is the index of refraction at the runway surface and* $a = 1.5 \times 10^{-6}$ m^{-1}. *That person's eyes are at a height* $h = 1.7$ m *above the runway. We have to find the horizontal distance d that person sees for the runway.*



Solution:

We will set up a differential equation that describes the change in angle of refraction with height in a medium in which refractive index of the medium is continuously changing with height.

Let $n(y)$ be the refractive index of the medium at height y; see the figure. According to Snell's law, we have

$$n(y)\sin\theta(y) = n(y+\Delta y)\sin\theta(y+\Delta y),$$

or

$$n(y)\sin\theta(y) = \left(n(y) + \frac{dn(y)}{dy}\Delta y\right) \times \left(\sin\theta(y) + \cos\theta(y)\frac{d\theta(y)}{dy}\Delta y\right).$$

Retaining terms up to first order in Δy, we get

$$n(y)\frac{\cos\theta(y)}{\sin\theta(y)}\frac{d\theta(y)}{dy} = -\frac{dn(y)}{dy},$$

or

$$\frac{d\left(n(y)\sin\theta(y)\right)}{dy} = 0.$$

The solution of this differential equation is

$$n(y)\sin\theta(y) = const.$$

It is given that the refractive index of the medium is given by the function

$$n(y) = n_0(1+ay).$$

Using the boundary condition

$$\theta(0) = \frac{\pi}{2},$$

for the ray that is tangential to the runway and passes by the person at the height of the eyes.

Thus the light ray is fixed by the requirement that the value of the constant in the equation

$$n(y)\sin\theta(y) = const$$

is

$$const = n_0.$$

The variation of the angle of refraction along the ray is given by the function

$$\sin\theta(y) = \frac{n_0}{n_0(1+ay)}.$$

The angle $\theta(h)$ of the ray from the normal at height *h* of the observer is

$$\sin\theta(h) = \frac{1}{1+ah}.$$

We are given that

$a = 1.5\times10^{-6}$ m^{-1}, and $h = 1.7$ m, this gives

$$\theta(h) = \sin^{-1}\left(\frac{1}{1+1.5\times1.7\times10^{-6}}\right) = 89.87^0.$$

As

$$\frac{d}{h} = \tan\theta(h),$$

we find that

$$d = 1.7 \times \tan\left(89.87^0\right) \text{ m} = 752.7 \text{ m}.$$

The length of the runway seen by the person is 752.7 m.