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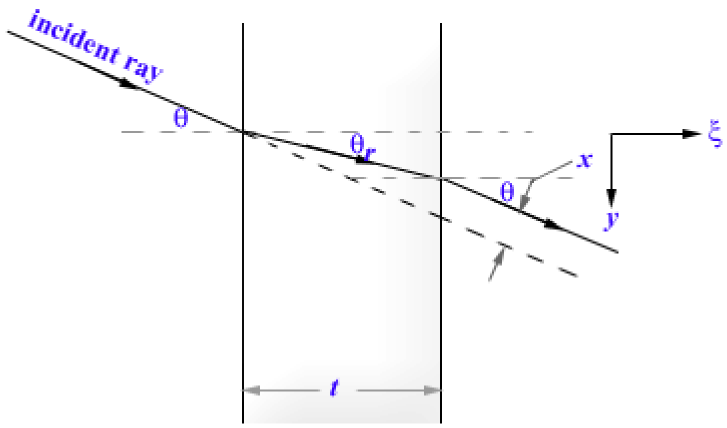
Problem 43.15 (RHK)

We have to prove that a ray of light incident on the surface of a sheet of plate glass of thickness t emerges from the opposite face parallel to its initial direction but displaced sideways as shown in the figure. (a) *We have to show that, for small angles of incidence* θ*, this displacement is given by*

$$x = t\theta \frac{n-1}{n} \ ,$$

where n is the index of refraction and θ *is measured in radians*. (b) We *have to calculate the displacement at a* 10^0 *angle of incidence through a* 1.0-cm *thick sheet of crown glass.*



Solution:

(a)

Let n be the refractive index of the plate glass. We assume that the angle of incidence θ is small and we can use the approximation

$$\sin\theta \;;\; \theta.$$

The angle of refraction θ_r is given by the Snell's law

$$n\sin\theta_r = \sin\theta,$$

and in the small angle approximation,

$$\theta_r \;;\; \frac{\theta}{n}.$$

Using the ξy-coordinate system shown in the figure, we can write for the equations of straight lines for the incident ray and the refracted rays the following equations:

$$y = (\tan\theta)\xi, \text{ (incident ray)}$$
$$y = (\tan\theta_r)\xi, \text{ (refracted ray)},$$

where ξ is the distance inside the plate glass measured from the edge where the incident ray meets the glass sheet. In the small angle approximation these equations can be written as

$$y = \theta\xi, \text{ (incident ray)},$$
$$y = \theta_r\xi, \text{ (refracted ray)}.$$

The thickness of the plate glass sheet is t. Therefore, the separation between the points where the refracted ray meets the opposite edge of the sheet and the point where the unrefracted incident ray would have touched the opposite edge of the sheet will be given by

$$d = t(\theta - \theta_r) = t\left(\theta - \frac{\theta}{n}\right) = t\theta\frac{(n-1)}{n}.$$

In the limit of small angles, the displacement x will also be approximately equal to d. Hence

$$x \approx d = t\theta\frac{(n-1)}{n}.$$

As the front and the back faces of the sheet are parallel, from the line diagram of the figure and the use of Snell's law for refraction from glass to air and from air to glass, we note that the ray will emerge from the glass sheet parallel to the incident ray.

(b)

Refractive index of crown glass, $n = 1.52$.

$$\theta = 10^0 = \frac{\pi}{180} \times 10 \text{ rad} = 0.174 \text{ rad}.$$

The thickness of the crown glass sheet, $t = 1.0$ cm.

Therefore, the displacement of the incident ray due to refraction in the glass sheet will be

$$x = 0.174 \times \frac{0.52}{1.52} \text{ cm} = 0.59 \text{ mm}.$$